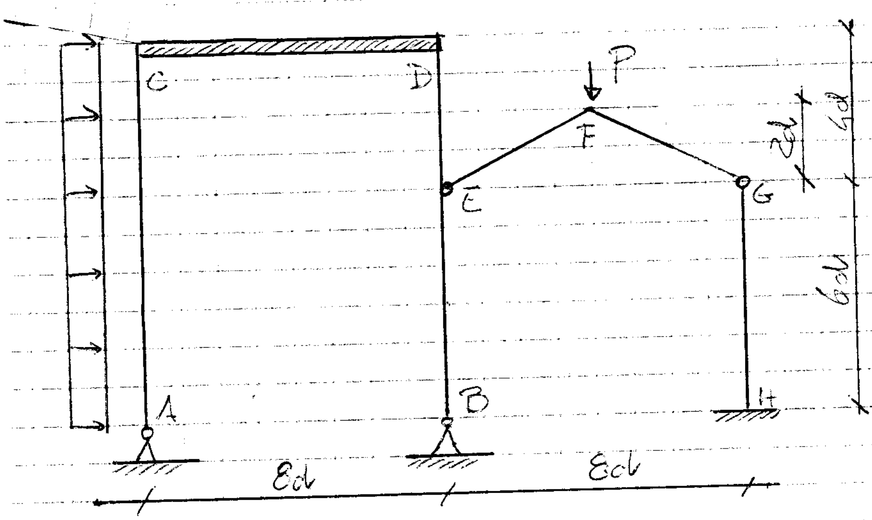
1



$d = 1\,m$

$q = 30\,kN/m$

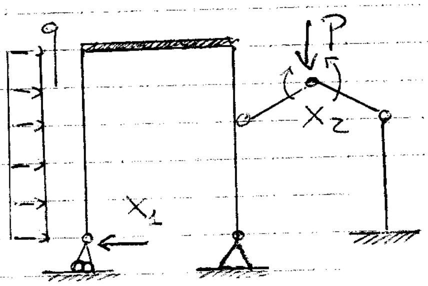
$P = 100\,kN = \frac{10}{3} qd$

$E = 210\,GPa$

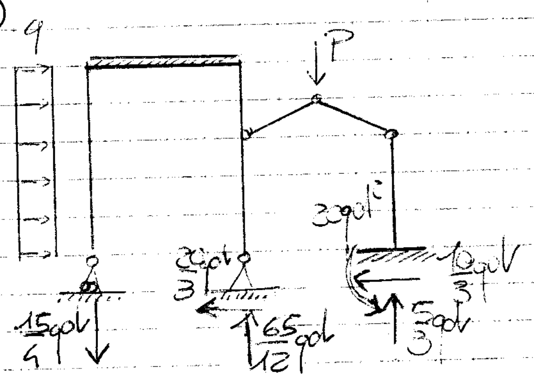
$\sigma_{amm} = 260\,MPa$

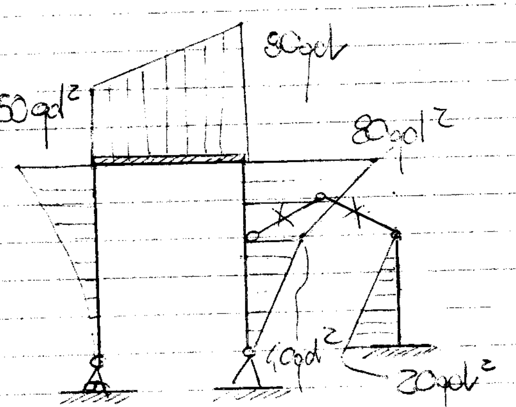
$\alpha = 1,2 \cdot 10^{-5}\,°C^{-1}$

a) Utilizzo il metodo delle forze

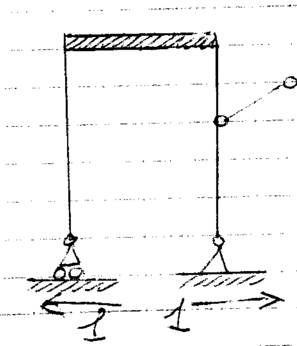


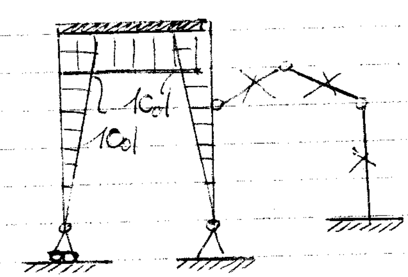
(0)



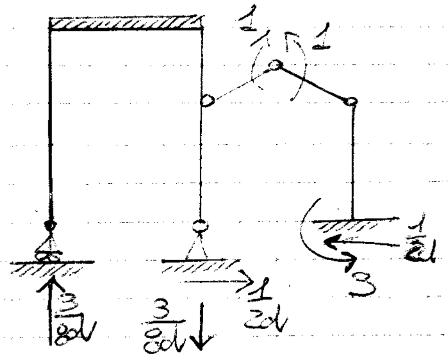


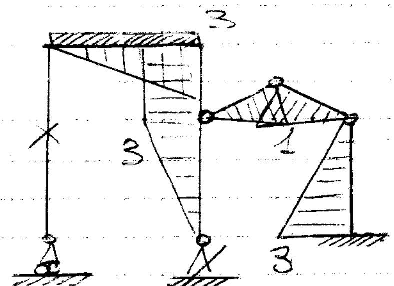
(1)





(2)





$$ES_{y_{11}} = 2\cdot\frac{1}{3}\cdot 10d\,(10d)^2 + (10d)^2\cdot 8d = \left(\frac{2}{3}\cdot 10 + 8\right)100d^3 = \left(\frac{20+24}{3}\right)100d^3 =$$

$$= \frac{44}{3}\cdot 100d^3 = \frac{4400}{3}d^3$$

$$ES_{y_{22}} = \frac{1}{\cancel{3}}\cdot 8d\cdot 3^{\cancel{2}} + \frac{1}{\cancel{3}}6d\cdot 3^{\cancel{2}} + \frac{1}{\cancel{3}}\cdot 6d\cdot 3^{\cancel{2}} + 3^2\cdot 4d + 2\cdot\frac{1}{3}(1)^2\,2\sqrt{5}\,d$$

$$= (24 + 18\cdot 2 + 36)d + \frac{4\sqrt{5}}{3}d = \left(96 + \frac{4\sqrt{5}}{3}\right)d$$

$$ES_{y_{12}} = -\frac{1}{\cancel{2}}\cdot\cancel{10}d^{5}\cdot 3\cdot 8d + \frac{1}{\cancel{3}}6d\cdot\cancel{3}\cdot 6d + \int_0^{4d}(6d+z)\,3\,dz$$

$$= 120d^2 + 36d^2 + \left[3\left(6dz + z^2/2\right)\right]_0^{4d} = 156d^2 + 3\left(6d\cdot 4d + 8d^2\right)$$

$$= 156d^2 + 96d^2 = 252d^2$$

$$ES_{y_{10}} = \int_0^{10d} q\frac{z^2}{2}(-z)\,dz + \int_0^{8d}\left(-50qd^2 - \frac{15}{4}qdz\right)10d\,dz - \frac{1}{\cancel{3}}40qd^2\cdot\cancel{6d}\cdot 6d + \int_0^{4d}(-40qd^2 - 10qdz)$$

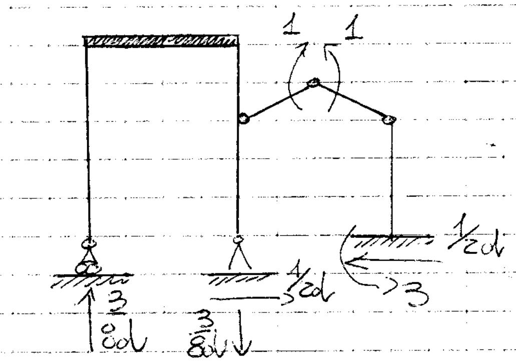
$$\cdot(6d+z)\,dz$$

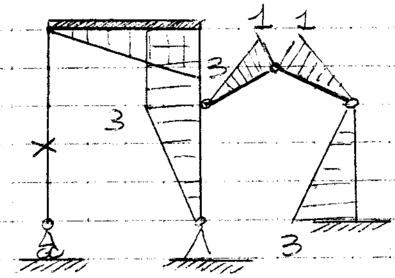
$$= \int_0^{10d} -q\frac{z^3}{2}dz + \int_0^{8d}\left(-500qd^3 - \frac{75}{2}qd^2z\right)dz - 480qd^4 + \int_0^{4d}\left(-240qd^3 - 40qd^2z - 60qd^2z - 10qdz^2\right)dz$$

$$= \left[-q\frac{z^4}{8}\right]_0^{10d} + \left[-500qd^3z - \frac{75}{4}qd^2z^2\right]_0^{8d} - 480qd^4 + \left[-240qd^3z - 50qd^2z^2 - \frac{10}{3}qdz^3\right]_0^{4d}$$

$$= -1250qd^4 - 4000qd^4 - 1200qd^4 - 480qd^4 - 960qd^4 - 800qd^4 - \frac{640}{3}qd^4$$

$$= -\left(8690 + \frac{640}{3}\right)qd^4$$





Calcolo dei coefficienti η_{ij}

$$EJ\eta_{11} = \frac{1}{3}(10d)^2\cdot 10d \cdot 2 = \frac{2000}{3}d^3$$

$$EJ\eta_{22} = \frac{1}{3}(3)^2\,6d\cdot 2 + (3)^2\cdot 4d + \frac{1}{3}(1)^2\,2\sqrt{5}d\,2 = \left(72+\frac{4\sqrt{5}}{3}\right)d$$

$$EJ\eta_{12} = \frac{1}{\cancel{3}}(6d)\,6d\cdot\cancel{3} + \int_0^{4d}(6d+z)\,3\,dz = 36d^2 + 3\left[6dz+\frac{z^2}{2}\right]_0^{4d} = 36d^2+3(24d^2+8d^2) =$$

$$= 132\,d^2$$

$$EJ\eta_{10} = \int_0^{10d}\left(-q\frac{z^2}{2}\right) z\,dz + \int_0^{4d}(-40qd^2-10qdz)(6d+z)\,dz - \frac{1}{\cancel{3}}6d\cdot 40qd^2\cdot 6d =$$

$$= \left[-q\frac{z^4}{8}\right]_0^{10d} + \int_0^{4d}(-240qd^3-40qd^2z-60qd^2z-10qdz^2)\,dz - 480qd^4 =$$

$$= -1250qd^4 - \left[240qd^3z - 50qd^2z^2 - \frac{10}{3}qdz^3\right]_0^{4d} - 480qd^4 =$$

$$= -1250qd^4 - 960qd^4 - 800qd^4 - \frac{640}{3}qd^4 - 480qd^4 = -\left(3490+\frac{640}{3}\right)qd^4$$

$$EJ\eta_{20} = \frac{1}{\cancel{3}}20qd^2\cdot\cancel{3}\cdot 6d - \frac{1}{\cancel{3}}40qd^2\,\cancel{3}\cdot 6d - \int_0^{4d}(+40qd^2+10qdz)\,3\,dz =$$

$$= 120qd^3 - 240qd^3 - 3\left[40qd^2z + 5qdz^2\right]_0^{4d} =$$

$$= -120qd^3 - 480qd^3 - 240qd^3 = -840qd^3$$

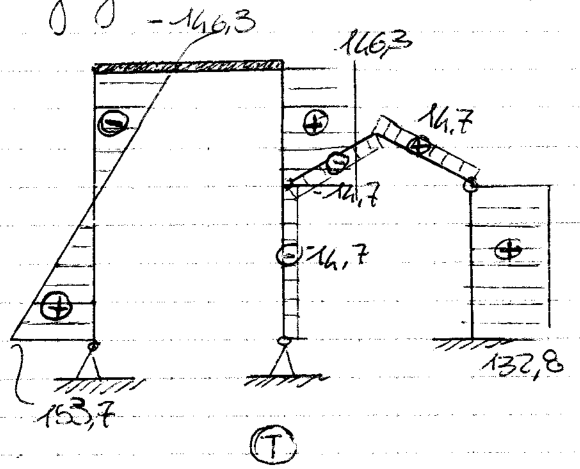
Sistema finale

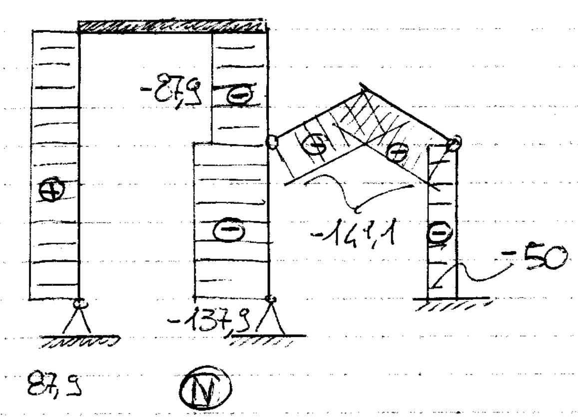
$$\begin{bmatrix} \frac{2000}{3}d^3 & 132d^2 \\ 132d^2 & \left(72+\frac{4\sqrt{5}}{3}\right)d \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} \left(3490+\frac{640}{3}\right)qd^4 \\ 840\,qd^3 \end{bmatrix}$$

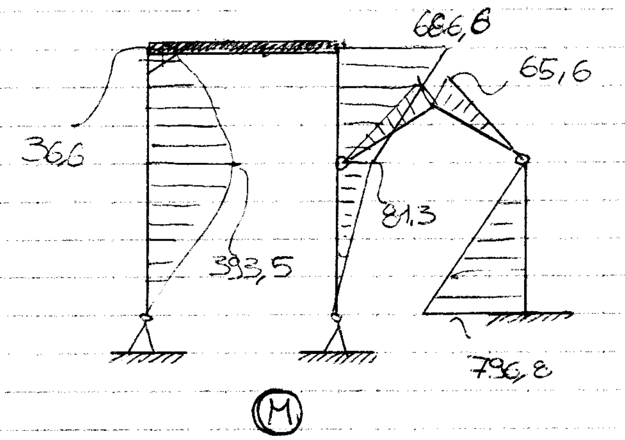
$X_1 = 153{,}7$ KN

$X_2 = 65{,}6$ KNm

Ⓑ Grafici dell'azione interna (i valori sono espressi in KN e KNm)







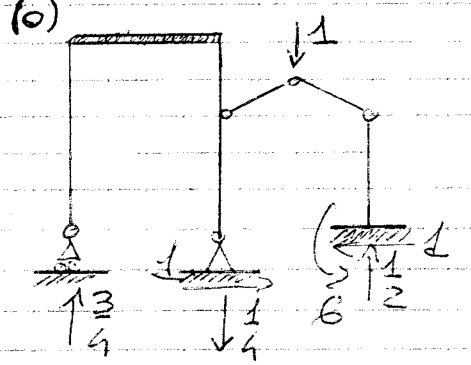
Ⓒ Progetto della struttura:

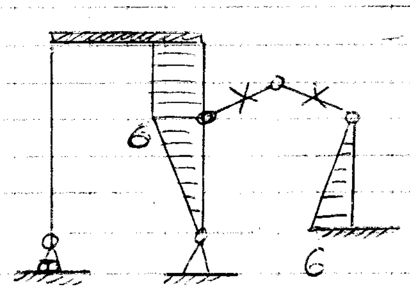
$$W_{min} = \frac{M_{max}}{\sigma_{amm}} = \frac{796,8 \cdot 10^3}{260} = 3064,6 \text{ cm}^3$$

Non posso usare sezioni rettangolari, quindi utilizzo una sezione HEB 450.

Ⓓ Per il calcolo dello spostamento in F applico una forza unitaria verticale in F e risolvo la struttura. Siccome il sistema (1) e (2) sono uguali a quelli calcolati al punto ⓐ, calcolo solamente le sollecitazioni nel sistema 0

(0)





$$-\eta_{10} = \frac{1}{3} 6 (6d)^2 + \int_0^{4d} 6(6d+z)\,dz = 72d^2 + 6\left(6d \cdot 4d + \frac{16d^2}{2}\right) = 264d^2$$

$$\eta_{20} = \frac{2(6d)\cancel{3}\cdot 6}{\cancel{3}} + 4d \cdot 6 \cdot 3 = 144d$$

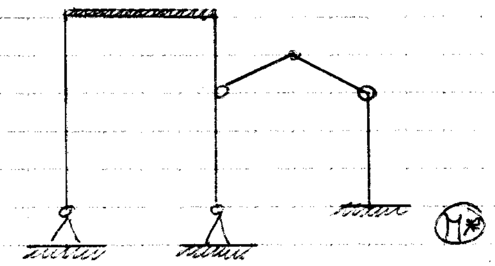
Sistema risolvente

$$\begin{bmatrix} \frac{2000}{3}d^3 & 132\,d^2 \\ 132\,d^2 & \left(72+\frac{4\sqrt{5}}{3}\right)d \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} -204\,d^2 \\ -144\,d \end{bmatrix}$$

$X_1 = -0{,}024$ KN

$X_2 = -1{,}878$ KNm

Da cui si ha il seguente grafico del momento



A questo punto applicando il PLV è possibile andare al calcolo dello spostamento in F

$$v_F = \int_{\Omega} M^* \cdot \frac{M}{EJ}\, dz$$

dove M è il momento flettente calcolato al punto Ⓑ.

Ⓔ Per considerare il carico Termico T_0 sarà necessario calcolare i coefficienti η^T_{10} e η^T_{20}:

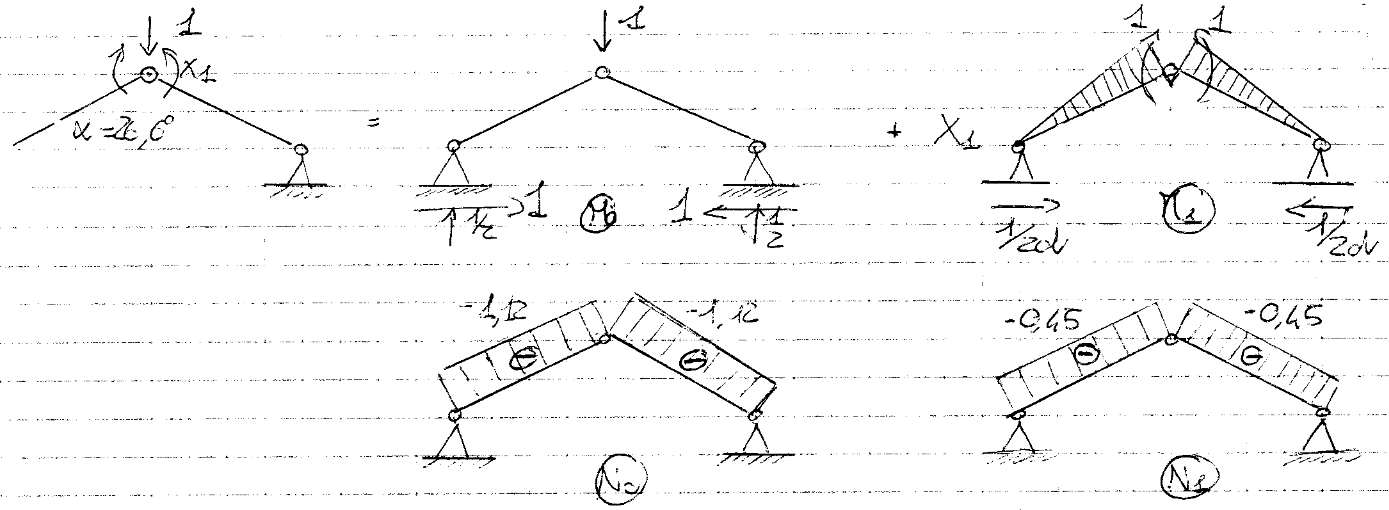
$$\eta^T_{10} = \int_0^{2\sqrt{5}d} N^{①}_{EF}\, \varepsilon^T_{EF}\, dz + \int_0^{2\sqrt{5}d} N^{①}_{FG}\, \varepsilon^T_{FG}\, dz = \dots = 2\sqrt{5}d\left(N^{①}_{EF} + N^{①}_{FG}\right)\alpha T_0$$

$$\eta^T_{20} = \int_0^{2\sqrt{5}d} N^{②}_{EF}\, \varepsilon^T_{EF}\, dz + \int_0^{2\sqrt{5}d} N^{②}_{FG}\, \varepsilon^T_{FG}\, dz = \dots = 2\sqrt{5}d\left(N^{②}_{EF} + N^{②}_{FG}\right)\alpha T_0$$

In questo modo si ricava il nuovo sistema

$$\begin{bmatrix} \eta_{11} & \eta_{12} \\ \eta_{21} & \eta_{22} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} -\eta_{10} - \eta^T_{10} \\ -\eta_{20} - \eta^T_{20} \end{bmatrix}$$

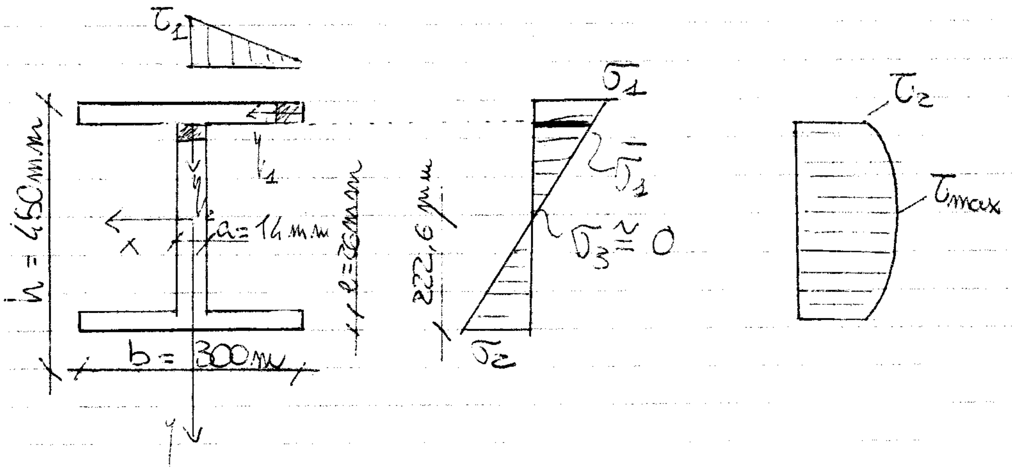
da cui è possibile ricavare i nuovi valori di X_1 e X_2.



1
X1
α = 26,6°
=
1
1/2
1
1
1/2
+ X1
1
1
-1,12
-1,12
-0,45
-0,45

F) Verifico la sezione in H:

$M = 796,8\ KNm$ $\quad N = -50KN$ $\quad T = 132,8\ KN$



$W = 3550\ cm^3$ $\quad J = 79887\ cm^4$ $\quad A = 218\ cm^2$

Tensioni normali

$$\sigma = \frac{N}{A} + \frac{M}{W} = \left(\frac{50 \cdot 10^3}{218 \cdot 10^{-4}} \pm \frac{768,8 \cdot 10^3}{3550 \cdot 10^{-6}}\right) \cdot 10^{-6} = \begin{cases} \sigma_1 = 219,14\ MPa \\ \sigma_2 = 214,55\ MPa \end{cases}$$

$\bar{\sigma}_1 = -194,08\ MPa$

Tensioni tangenziali

$$\tau_1 = \frac{TS_1}{eJ} = \frac{132,8 \cdot 10^3}{0,026 \cdot 79887 \cdot 10^{-8}} \cdot \left(\frac{300}{2} \cdot 26 \cdot 10^{-6}\right)\left(\frac{450}{2} - \frac{26}{2}\right) \cdot 10^{-3} \cdot 10^{-6} = 5,29\ MPa$$

$$\tau_2 = \frac{TS_2}{aJ} = \frac{132,8 \cdot 10^3}{0,014 \cdot 79887 \cdot 10^{-8}} \cdot \left(300 \cdot 26 \cdot 10^{-6}\right)\left(\frac{450}{2} - \frac{26}{2}\right) \cdot 10^{-3} \cdot 10^{-6} = 19,63\ MPa$$

$$\tau_{max} = \tau_2 + \frac{TS_3}{aJ} = 19,63 + \frac{132,8 \cdot 10^3}{0,014 \cdot 79887 \cdot 10^{-8}} \left(450 \cdot 14 \cdot 10^{-6}\right)\left(\frac{450}{2} - 26\right)\frac{1}{2} \cdot 10^{-3} \cdot 10^{-6} =$$

$= 27,08\ MPa$

Calcolo delle tensioni ideali:

$\sigma_{id}^1 = \sqrt{\sigma_1^2 + 3\tau_1^2} = 219,3\ MPa < \sigma_{amm}$

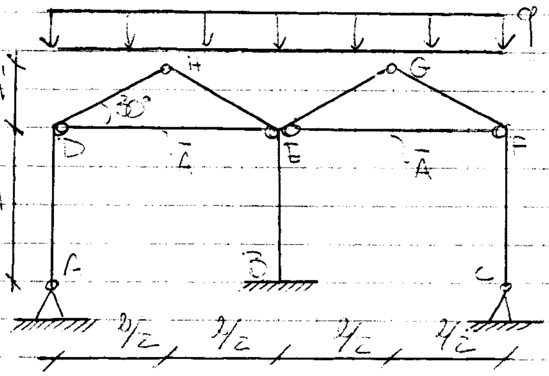
$\sigma_{id}^2 = \sqrt{\bar{\sigma}_1^2 + 3\tau_2^2} = 197,0\ MPa < \sigma_{amm}$

$\sigma_{id}^3 = \sqrt{\sigma_3^2 + 3\tau_3^2} = 46,90\ MPa < \sigma_{amm}$

La sezione risulta verificata.

[illegible]

Esercizio 2



$q = 20$ k/m $\quad \sigma_{am} = 240$ MPa

$l = 6$ m $\quad E = 210$ GPa

$h = 4$ m $\quad \alpha = 1{,}2 \cdot 10^{-5}\ °C^{-1}$

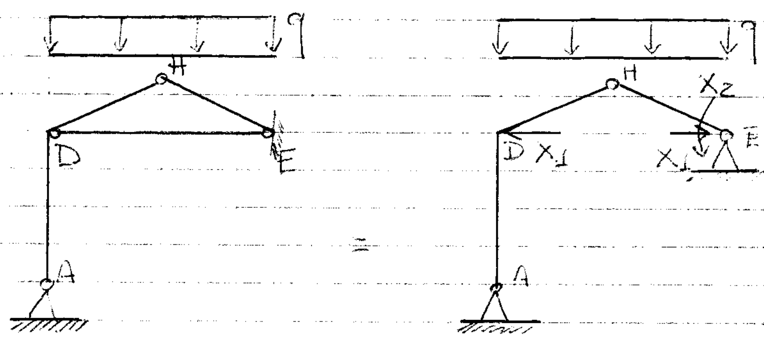
$\bar{A} = 10\ cm^2$

$h' = 1{,}732$ m

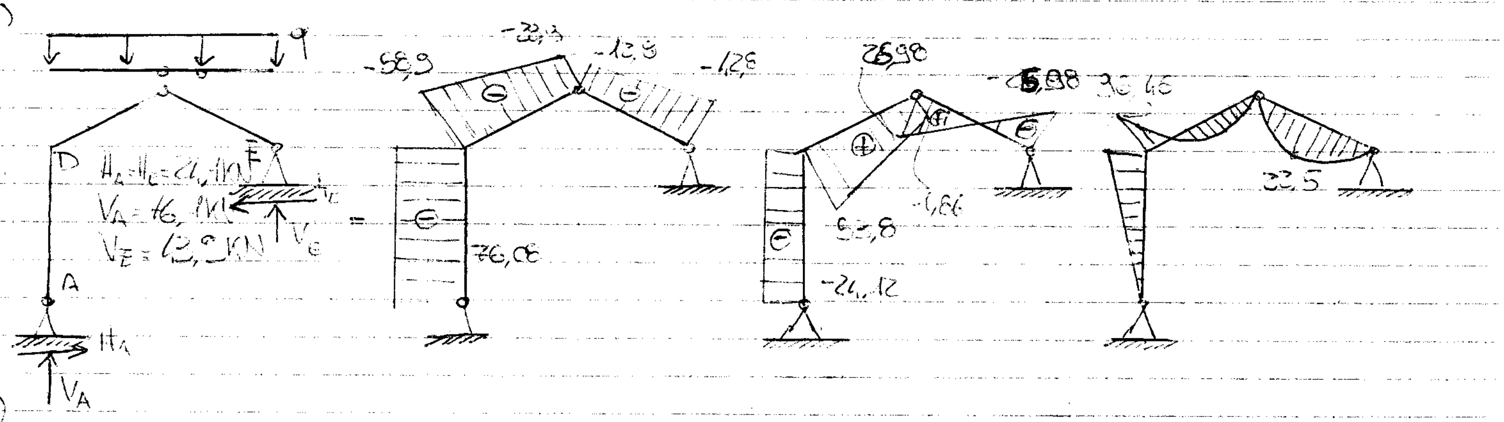
$l' = 3{,}464$ m

(a) Progettare a flessione con profili HEB la struttura, trascurando le deformazioni assiali in tutte le aste

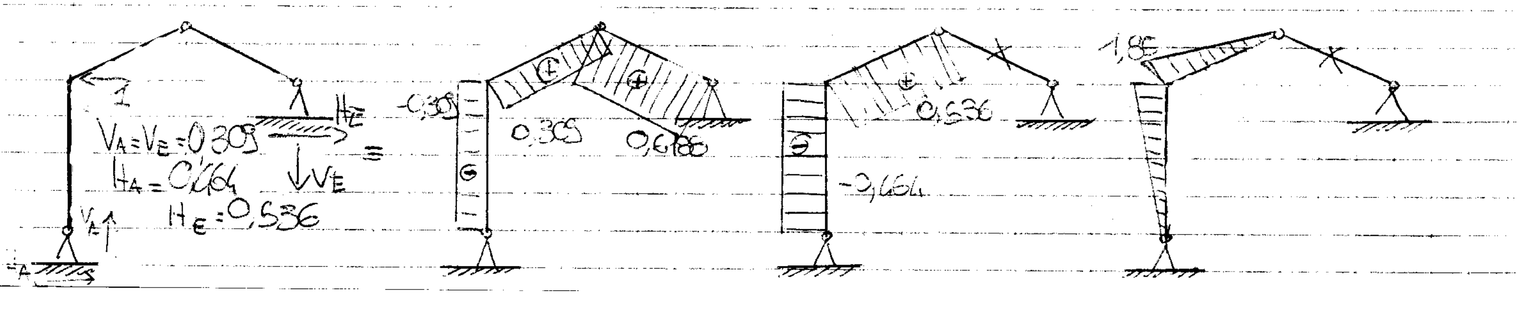
La struttura in esame è simmetrica di geometria, vincoli e carichi, perciò posso studiarne solo metà. In particolare, data la simmetria del problema e l'infinita rigidezza assiale del tratto BE, utilizzo il seguente schema



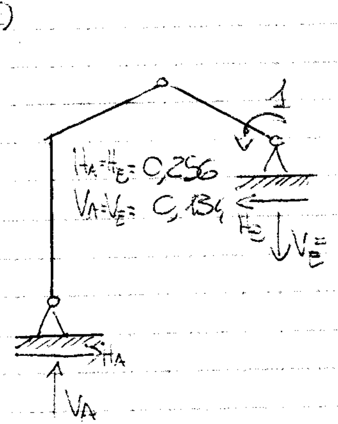
(0)

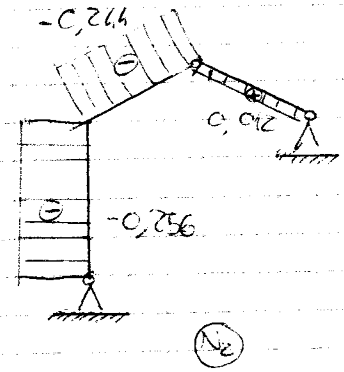


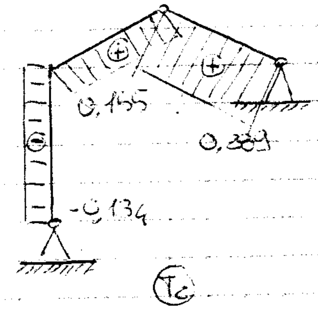
(1)

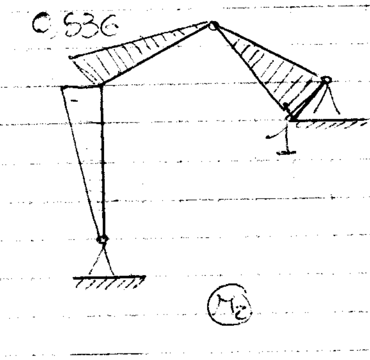


(2)









$$\eta_{11} = \int \frac{M_1^2}{EJ}\,dz = \frac{1{,}85^2}{3EJ}\left[4+3{,}464\right] = \frac{8{,}52}{EJ}$$

$$\eta_{22} = \int \frac{M_2^2}{EJ}\,dz = \frac{1}{3EJ}\left[0{,}536^2(4+3{,}464) + 1^2\cdot 3{,}464\right] = \frac{1{,}87}{EJ}$$

$$\eta_{12} = \int \frac{M_1 M_2}{EJ}\,dz = \frac{1}{3EJ}\; 1{,}85\cdot 0{,}536\cdot(4+3{,}464) = \frac{2{,}47}{EJ}$$

$$\eta_{10} = \int \frac{M_1 M_0}{EJ}\,dz = \frac{1}{EJ}\left[\frac{1}{3}\cdot 1{,}85\cdot 96{,}46\cdot 4 + \int_0^{l'} (-1{,}85+0{,}536z)\left(-96{,}46+53{,}8z-17{,}32\cdot\frac{z^2}{2}\right)dz\right]$$

$$= \frac{1}{EJ}\left[237{,}93 + 165{,}24\right] = \frac{403{,}17}{EJ}$$

$$\eta_{20} = \int \frac{M_2 M_0}{EJ}\,dz = \frac{1}{EJ}\left[\frac{1}{3}\cdot 0{,}536\cdot 96{,}46\cdot 4 + \int_0^{l'}(-0{,}536+0{,}155\cdot z)\left(-96{,}46+53{,}8z-17{,}32\cdot\frac{z^2}{2}\right)dz\right.$$

$$\left. + \int_0^{l'}(1-0{,}289z)\left(25{,}98z-17{,}32\cdot\frac{z^2}{2}\right)dz\right]$$

$$= \frac{1}{EJ}\left[68{,}94+47{,}92+21{,}95\right] = \frac{138{,}81}{EJ}$$

Sistema finale

$$\begin{bmatrix} 8{,}52 & 2{,}47 \\ 2{,}47 & 1{,}87 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} -403{,}17 \\ -138{,}81 \end{bmatrix} \Rightarrow \begin{aligned} X_1 &= -41{,}81\ \text{KN} \\ X_2 &= -19\ \text{KNm} \end{aligned}$$

Grafici finali