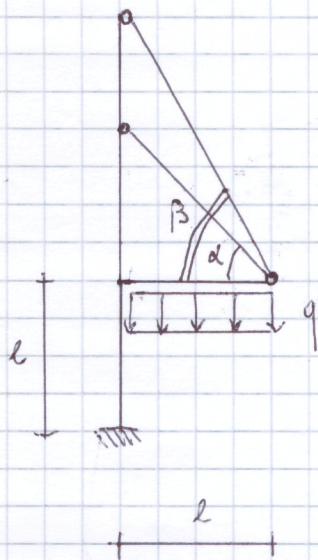


SOLUZIONE COMPITO 21/02/2022



$$l = 4 \text{ m}$$

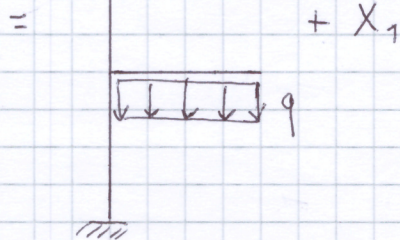
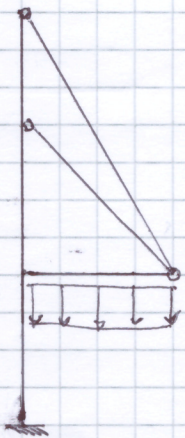
$$q = 3000 \text{ N/m}$$

$$E = 210000 \text{ MPa}$$

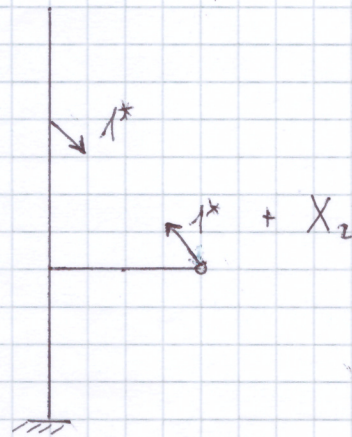
$$\sigma_{\text{amm}} = 190 \text{ MPa}$$

2 volte spostata

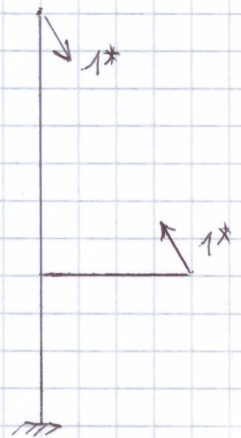
1)



SISTEMA 0

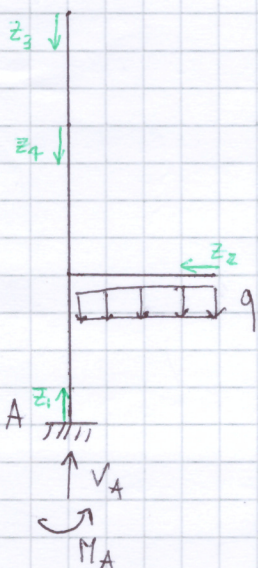


SISTEMA 1



SISTEMA 2

SISTEMA 0



$$\uparrow) V_A = ql = 12000 \text{ N}$$

$$\rightarrow) H_A = 0$$

$$\curvearrow) M_A = q \frac{l}{2} \cdot l = 24000 \text{ N/m}$$

$$N(z_1) = -12000 \text{ N}$$

$$N(z_2) = 0$$

$$N(z_3) = 0$$

$$N(z_4) = 0$$

$$T(z_1) = 0$$

$$T(z_2) = qz$$

$$T(z_3) = 0$$

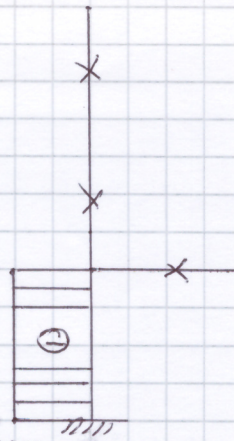
$$T(z_4) = 0$$

$$M(z_1) = -q \frac{z^2}{2} = -24000 \text{ Nm}$$

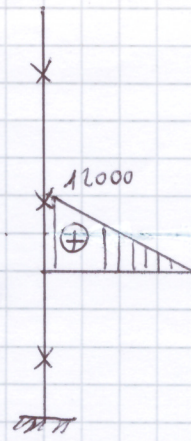
$$M(z_2) = -\frac{qz^2}{2}$$

$$M(z_3) = 0$$

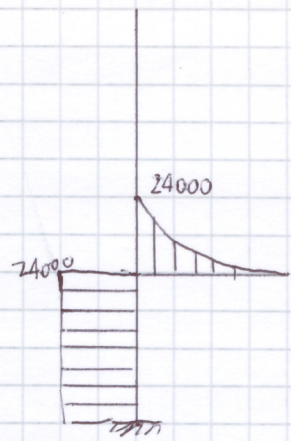
$$M(z_4) = 0$$



N [N]

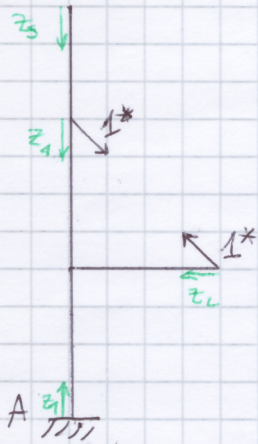


T [N]



M [Nm]

SISTEMA 1



$$\uparrow V_A = 0$$

$$\rightarrow H_A = 0$$

$$\curvearrowright M_A = 0$$

$$1^*_1 = \sqrt{2}/2 = 1''$$

$$N(z_1) = 0$$

$$N(z_2) = -\sqrt{2}/2$$

$$N(z_3) = 0$$

$$N(z_4) = -\sqrt{2}/2$$

$$T(z_1) = 0$$

$$T(z_2) = -\sqrt{2}/2$$

$$T(z_3) = 0$$

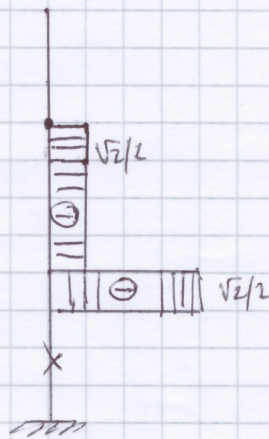
$$T(z_4) = \sqrt{2}/2$$

$$M(z_1) = 0$$

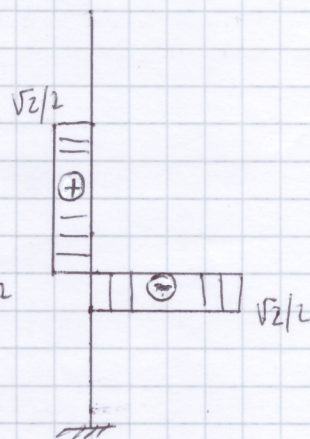
$$M(z_2) = \frac{\sqrt{2}}{2} z$$

$$M(z_3) = 0$$

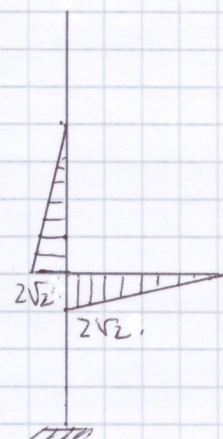
$$M(z_4) = -\frac{\sqrt{2}}{2} z$$



N [N]

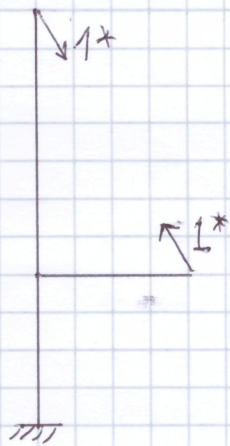


T [N]



M [Nm]

SISTEMA 2



$$\begin{aligned} \uparrow) V_A &= 0 \\ \rightarrow) H_A &= 0 \\ \curvearrowright) M_A &= 0 \end{aligned}$$

$$\begin{aligned} \begin{matrix} \nearrow 1_L \\ \nwarrow 1_{II} \end{matrix} & \quad 1_{II} = \frac{1}{2} \quad 1_L = \frac{\sqrt{3}}{2} \end{aligned}$$

$$N(z_1) = 0$$

$$N(z_2) = -1/2$$

$$N(z_3) = -\sqrt{3}/2$$

$$N(z_4) = -\sqrt{3}/2$$

$$T(z_1) = 0$$

$$T(z_2) = -\sqrt{3}/2$$

$$T(z_3) = 1/2$$

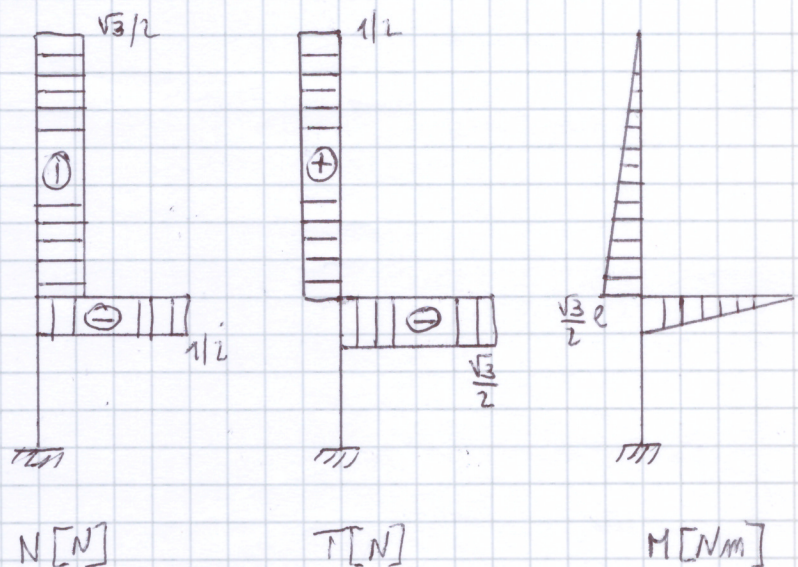
$$T(z_4) = 1/2$$

$$M(z_1) = 0$$

$$M(z_2) = \frac{\sqrt{3}}{2} z$$

$$M(z_3) = -\frac{1}{2} z$$

$$M(z_4) = -\frac{1}{2}(\sqrt{3}-1)l - \frac{1}{2} z$$



$$\eta_{10} = \frac{1}{EJ} \left[\int_0^4 \left(-\frac{qz^2}{2} \right) \left(\frac{\sqrt{2}}{2} z \right) dz \right] = -\frac{67882}{EJ}$$

$$\eta_{11} = \frac{1}{EJ} \left[2 \int_0^4 \left(\frac{\sqrt{2}}{2} z \right)^2 dz \right] = \frac{64}{3EJ}$$

$$\eta_{12} = \frac{1}{EJ} \left[\int_0^4 \left(\frac{\sqrt{2}}{2} z \right) \left(\frac{\sqrt{3}}{2} z \right) dz + \int_0^4 \left(-\frac{1}{2} [(\sqrt{3}-1)l + z] \right) \left(-\frac{\sqrt{2}}{2} z \right) dz \right] = \frac{72l}{25EJ}$$

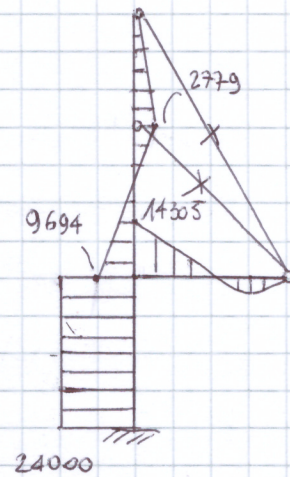
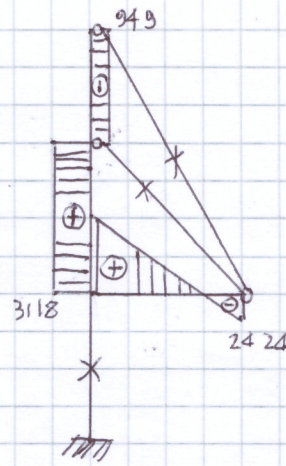
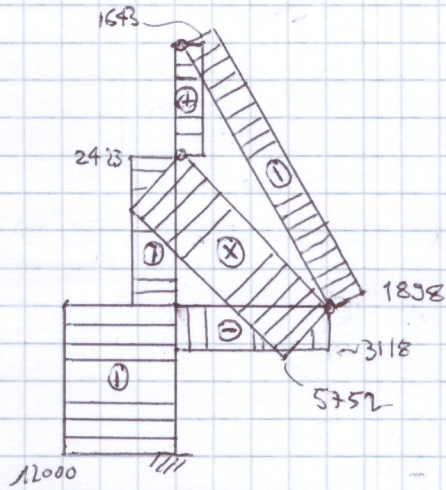
$$\eta_{21} = \frac{1}{EJ} \left[\int_0^4 \left(\frac{\sqrt{3}}{2} z \right)^2 dz + \int_0^{\sqrt{3}l} \left(-\frac{1}{2} z \right)^2 dz \right] = \frac{43.71}{EJ}$$

$$\eta_{20} = \frac{1}{EJ} \left[\int_0^4 \left(-\frac{qz^2}{2} \right) \left(\frac{\sqrt{3}}{2} z \right) dz \right] = -\frac{83738}{EJ}$$

$$\begin{vmatrix} \eta_{11} & \eta_{12} \\ \eta_{12} & \eta_{21} \end{vmatrix} \begin{vmatrix} X_1 \\ X_2 \end{vmatrix} = \begin{vmatrix} -\eta_{10} \\ -\eta_{20} \end{vmatrix}$$

$$X_1 = 5752 \text{ N}$$

$$X_2 = -1898 \text{ N}$$



N [N]

T [N]

M [N·m]

2) Progetto con profilo IPE

$$M_{\max} = 24000 \text{ N·m} \quad N = 12000 \text{ N}$$

$$W_{\min} = \frac{24000 \cdot 1000}{190} = 126315 \text{ mm}^3 = 126.3 \text{ cm}^3$$

$$\text{ADOTTO IPE 180} \rightarrow A = 23.9 \text{ cm}^2 \quad W_x = 146 \text{ cm}^3 \quad J_x = 1317 \text{ cm}^4$$

$$\text{Verifica} \rightarrow \sigma_z = \frac{N}{A} \pm \frac{M}{W} = -\frac{12000}{2390} \pm \frac{24000 \cdot 10^3}{146 \cdot 10^3}$$

$$= -5.02 \pm 164.38 < \begin{matrix} -169.4 \text{ V} \\ 159.36 \text{ V} \end{matrix}$$

3) Deformabilità annale delle travi

$$\eta_{11}^{\text{trave}} = \int_0^{\sqrt{2}l} \frac{1}{EA} dz = \frac{\sqrt{2}l}{EA} = \frac{4\sqrt{2}}{EA}$$

$$\rightarrow \eta_{11}^{\text{NSM}} = \eta_{11} + \eta_{11}^{\text{trave}}$$

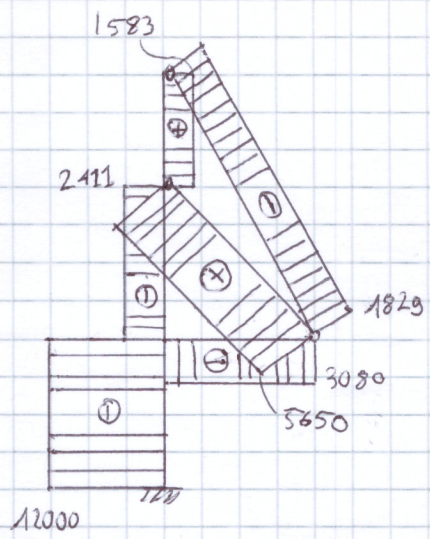
$$\eta_{22}^{\text{trave}} = \int_0^{2l} \frac{1}{EA} dz = \frac{2l}{EA} = \frac{8}{EA}$$

$$\rightarrow \eta_{22}^{\text{NSM}} = \eta_{22} + \eta_{22}^{\text{trave}}$$

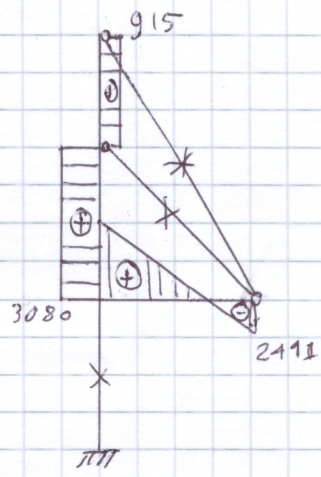
$$\begin{bmatrix} \eta_{11}^{\text{NSM}} & \eta_{12} \\ \eta_{12} & \eta_{22}^{\text{NSM}} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} -\eta_{10} \\ -\eta_{20} \end{bmatrix}$$

$$X_1 = 5650 \text{ N}$$

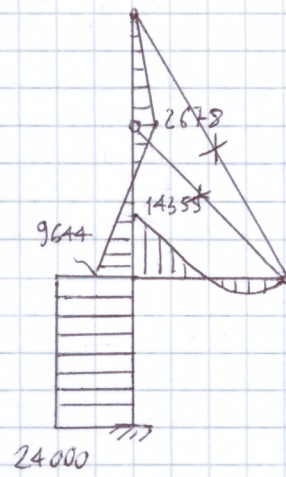
$$X_2 = -1829$$



$N[N]$



$T[N]$



$M[Nm]$