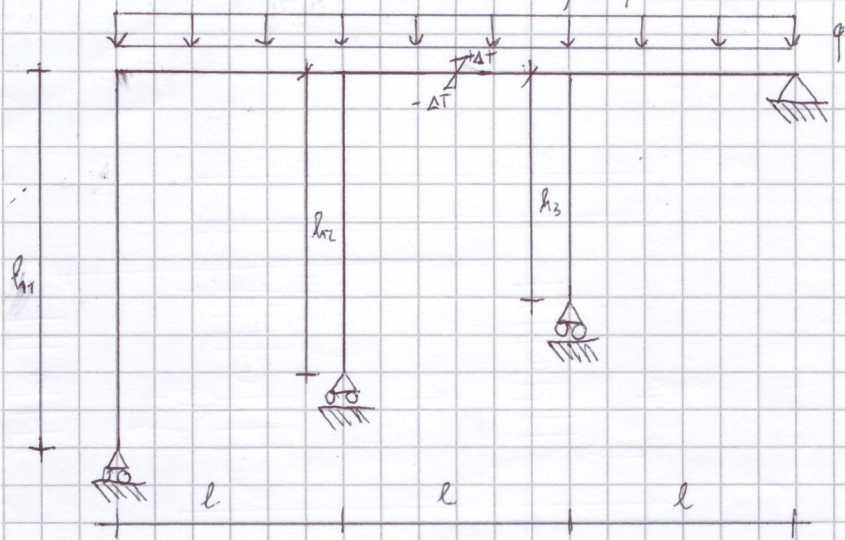


SOLUZIONE COMPITO 09/09/2022



$$q = 3000 \text{ N/m}$$

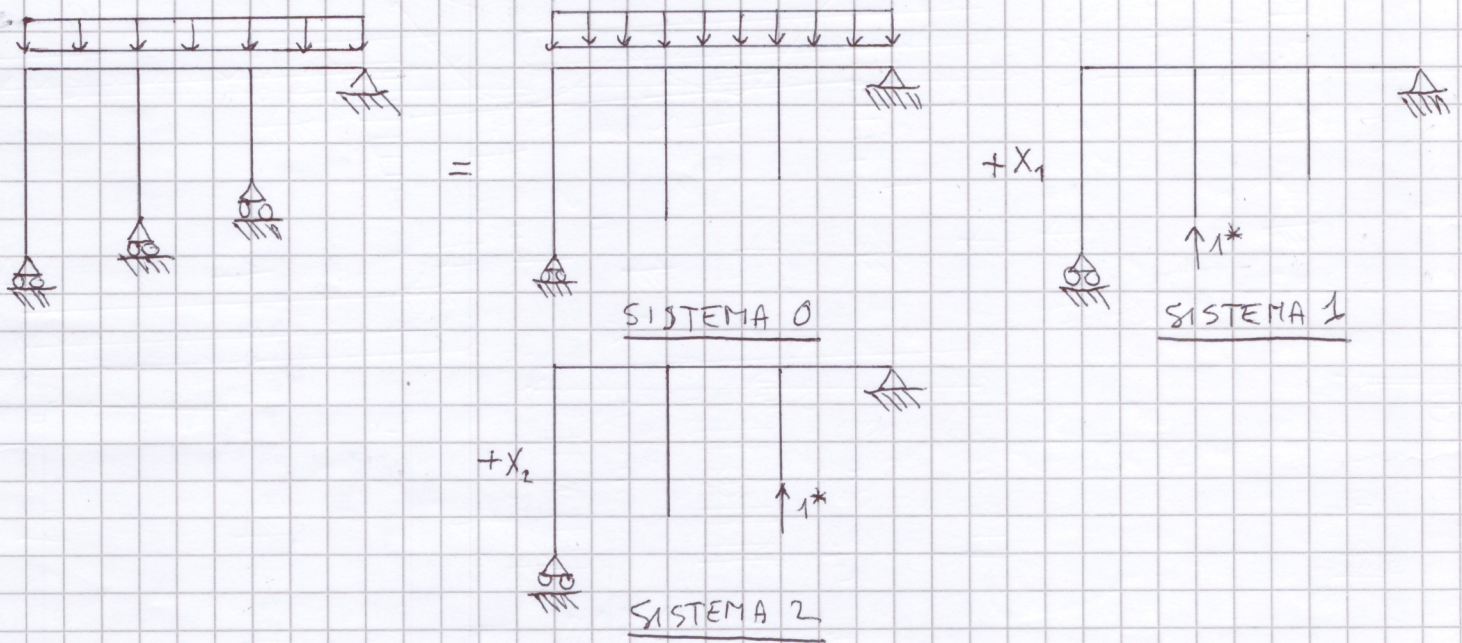
$$l = 3 \text{ m}$$

$$h_1 = 5 \text{ m}$$

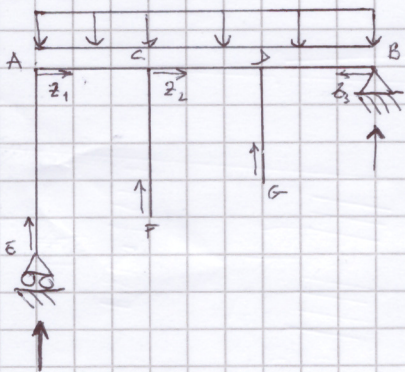
$$h_2 = 4 \text{ m}$$

$$h_3 = 3 \text{ m}$$

1) Trascurare carico termico e deformazione assiale



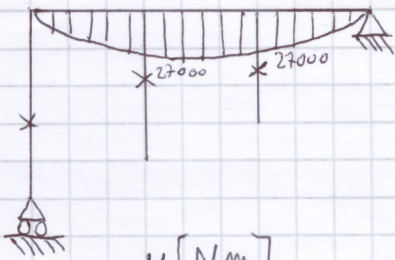
SISTEMA 0



$$\rightarrow H_B = 0$$

$$\uparrow V_E + V_B = 3ql \rightarrow V_B = \frac{3}{2}ql$$

$$\curvearrowright V_E = \frac{3}{2}ql$$

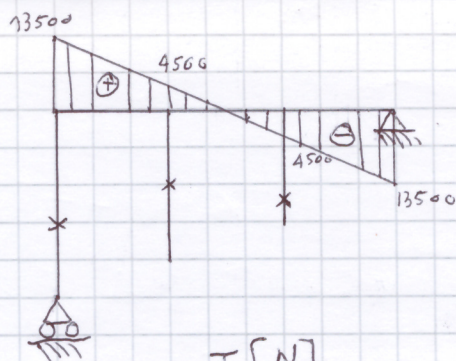


$M [Nm]$

$$M(z_1) = \frac{3}{2} q l z - q \frac{z^2}{2}$$

$$M(z_2) = q l^2 - q \frac{z^2}{2} + \frac{1}{2} q l z$$

$$M(z_3) = \frac{3}{2} q l z - q \frac{z^2}{2}$$

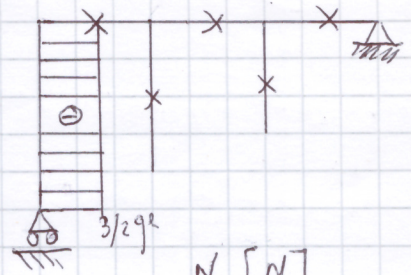


$T [N]$

$$T(z_1) = \frac{3}{2} q l - q z$$

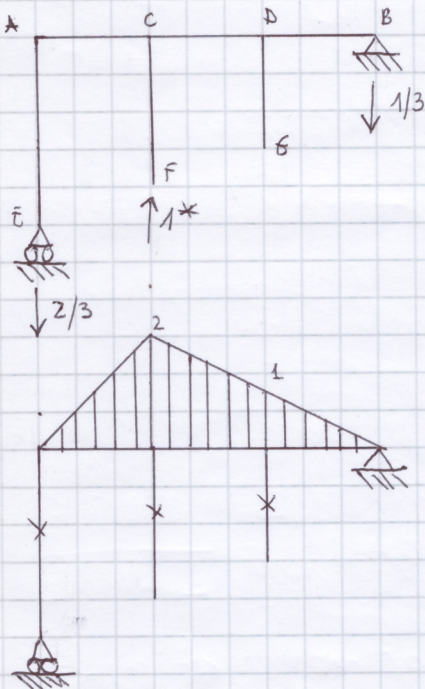
$$T(z_2) = \frac{1}{2} q l - q z$$

$$T(z_3) = +q z - \frac{3}{2} q l$$



$N [N]$

SISTEMA 1



$M [Nm]$

$$M(z_1) = -\frac{2}{3} z$$

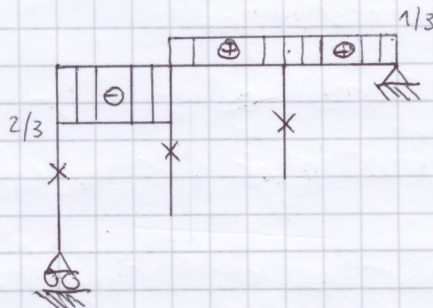
$$M(z_2) = -z + \frac{1}{3} z$$

$$M(z_3) = -\frac{1}{3} z$$

$$\rightarrow) H_B = 0$$

$$\uparrow) V_B + V_E = 1 \rightarrow V_B = \frac{1}{3}$$

$$B) V_E = \frac{2}{3}$$

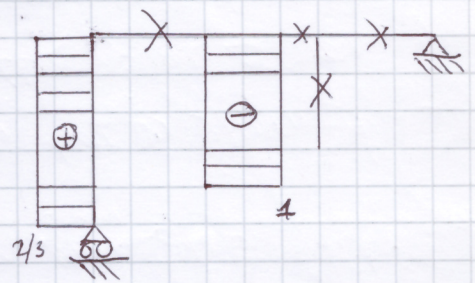


$T [N]$

$$T(z_1) = -\frac{2}{3}$$

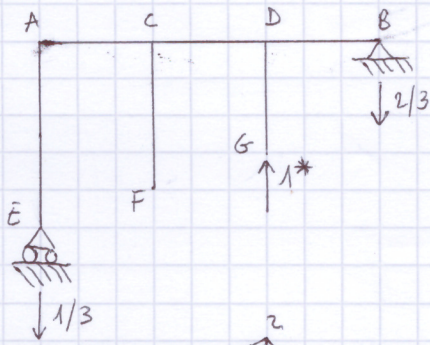
$$T(z_2) = \frac{1}{3}$$

$$T(z_3) = 1/3$$



$N [N]$

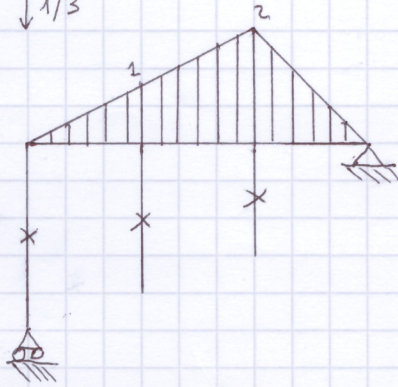
SISTEMA 2



$$\rightarrow) H_B = 0$$

$$\uparrow) V_B + V_E = 1 \rightarrow V_B = 2/3$$

$$B \uparrow) V_E = 1/3$$

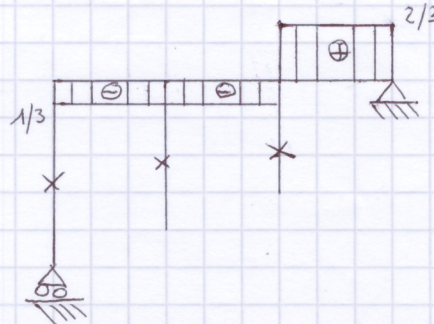


M [Nmm]

$$M(z_1) = -\frac{1}{3}z$$

$$M(z_2) = -1 - \frac{1}{3}z$$

$$M(z_3) = -\frac{2}{3}z$$

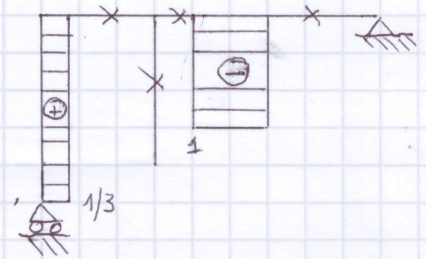


T [N]

$$T(z_1) = -\frac{1}{3}$$

$$T(z_2) = -\frac{1}{3}$$

$$T(z_3) = \frac{2}{3}$$



N [N]

$$\eta_{10} = \frac{1}{EJ} \left[\int_0^l \left(\frac{3}{2}qlz - q\frac{z^2}{2} \right) \left(-\frac{2}{3}z \right) dz + \int_0^l \left(ql^2 - q\frac{z^2}{2} + \frac{1}{2}qlz \right) \left(-2 + \frac{1}{3}z \right) dz + \int_0^l \left(\frac{3}{2}qlz - q\frac{z^2}{2} \right) \left(-\frac{1}{3}z \right) dz \right] = -\frac{222750}{EJ}$$

$$\eta_{11} = \frac{1}{EJ} \left[\int_0^l \left(-\frac{2}{3}z \right) dz + \int_0^l \left(-2 + \frac{1}{3}z \right)^2 dz + \int_0^l \left(-\frac{1}{3}z \right)^2 dz \right] = \frac{12}{EJ}$$

$$\eta_{12} = \frac{1}{EJ} \left[\int_0^l \left(-\frac{2}{3}z \right) \left(-\frac{1}{3}z \right) dz + \int_0^l \left(-2 + \frac{1}{3}z \right) \left(-1 - \frac{1}{3}z \right) dz + \int_0^l \left(-\frac{1}{3}z \right) \left(-\frac{2}{3}z \right) dz \right] = \frac{21}{2EJ}$$

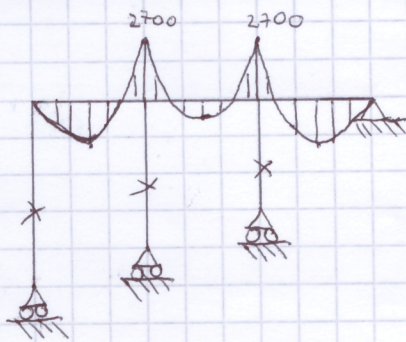
$$\eta_{20} = \frac{1}{EJ} \left[\int_0^l \left(\frac{3}{2}qlz - q\frac{z^2}{2} \right) \left(-\frac{1}{3}z \right) dz + \int_0^l \left(-1 - \frac{1}{3}z \right) \left(ql^2 - q\frac{z^2}{2} + \frac{1}{2}qlz \right) dz + \int_0^l \left(-\frac{2}{3}z \right) \left(\frac{3}{2}qlz - q\frac{z^2}{2} \right) dz \right] = -\frac{222750}{EJ}$$

$$\eta_{22} = \frac{1}{EJ} \left[\int_0^l \left(-\frac{1}{3}z \right)^2 dz + \int_0^l \left(-1 - \frac{1}{3}z \right)^2 dz + \int_0^l \left(-\frac{2}{3}z \right)^2 dz \right] = \frac{12}{EJ}$$

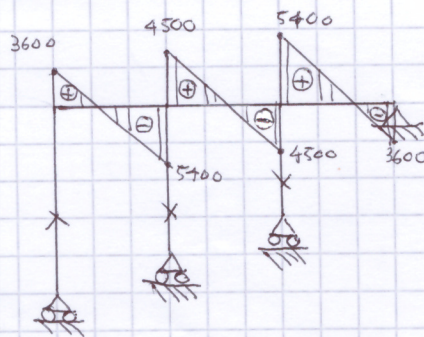
$$\begin{vmatrix} \eta_{11} & \eta_{12} \\ \eta_{12} & \eta_{22} \end{vmatrix} \begin{vmatrix} X_1 \\ X_2 \end{vmatrix} = \begin{vmatrix} -\eta_{10} \\ -\eta_{20} \end{vmatrix}$$

$$X_1 = 9900 \text{ N}$$

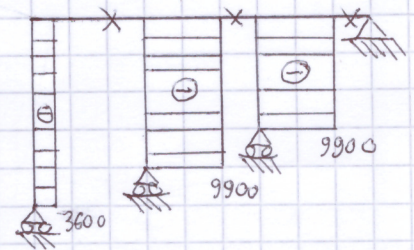
$$X_2 = 9900 \text{ N}$$



M [Nm]



T [N]



N [N]

2) Progetto e Verifica profilo

$$M_{\max} = 2700 \text{ Nm}$$

$$N = 0 \text{ N}$$

$$W_{\min} = \frac{2700 \cdot 1000}{190} = 14210.52 \text{ mm}^3 = 14.210 \text{ cm}^3$$

$$\text{Adatto IPE 80} \quad A = 7.64 \text{ cm}^2 \quad I_x = 80.14 \text{ cm}^4 \quad W_x = 20.03 \text{ cm}^3$$

$$\text{Verifica } \sigma = \frac{M}{W} = \frac{2700 \cdot 10^3}{20.03 \cdot 10^3} = 134 \text{ MPa} < 190 \text{ MPa} \quad \checkmark$$

3) Corico Termico

$$X_T = -\frac{2\alpha\Delta T}{h} = -\frac{3}{500}$$

$$\alpha = 1.2 \cdot 10^{-5} \text{ } ^\circ\text{C}^{-1}$$

$$\Delta T = 20^\circ\text{C}$$

$$h = 80 \text{ mm}$$

$$\left\{ \begin{aligned} \eta_{10} + \eta_{11} X_1 + \eta_{12} X_2 + \int M_1 X_T &= 0 \\ \eta_{20} + \eta_{21} X_1 + \eta_{22} X_2 + \int M_2 X_T &= 0 \end{aligned} \right.$$

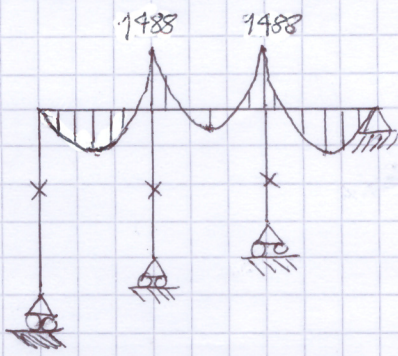
$$\left\{ \begin{aligned} \eta_{10} + \eta_{11} X_1 + \eta_{12} X_2 + \int M_1 X_T &= 0 \\ \eta_{20} + \eta_{21} X_1 + \eta_{22} X_2 + \int M_2 X_T &= 0 \end{aligned} \right.$$

$$\int M_1 X_T = \int_0^l \left(-\frac{2\alpha\Delta T}{h} \right) \left(-\frac{2}{3}z \right) dz + \int_0^l \left(-\frac{2\alpha\Delta T}{h} \right) \left(-l + \frac{1}{3}z \right) dz = \frac{27}{500}$$

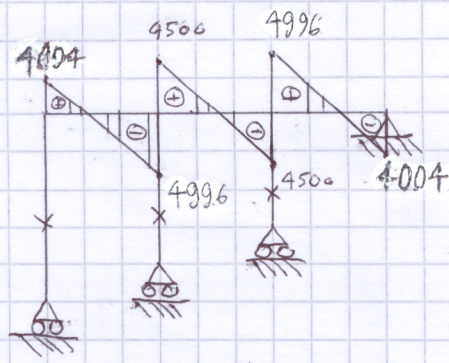
$$\int M_2 X_T = \int_0^l \left(-\frac{2\alpha\Delta T}{h} \right) \left(-\frac{1}{3}z \right) dz + \int_0^l \left(-\frac{2}{3}z \right) \left(-\frac{2\alpha\Delta T}{h} \right) dz = \frac{27}{500}$$

$$X_1 = 9496 \text{ N}$$

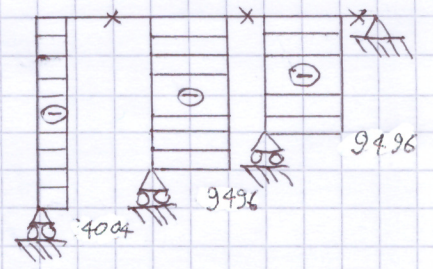
$$X_2 = 9496 \text{ N}$$



M [Nm]



T [N]



N [N]