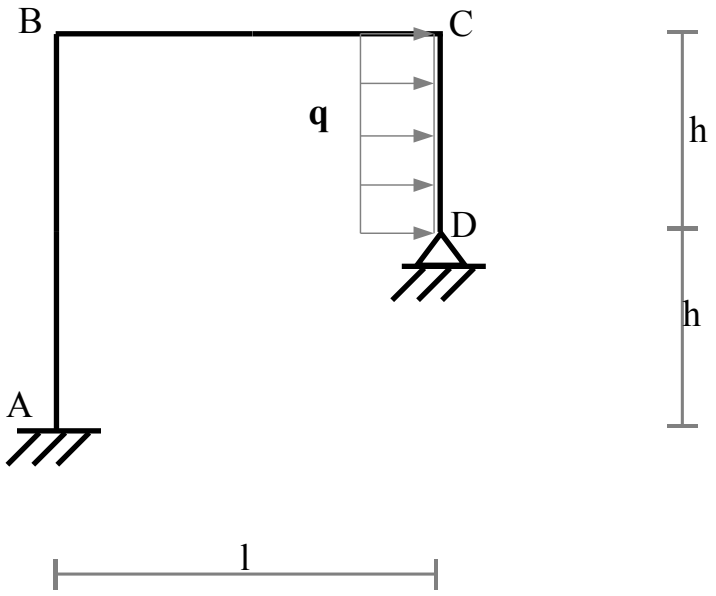
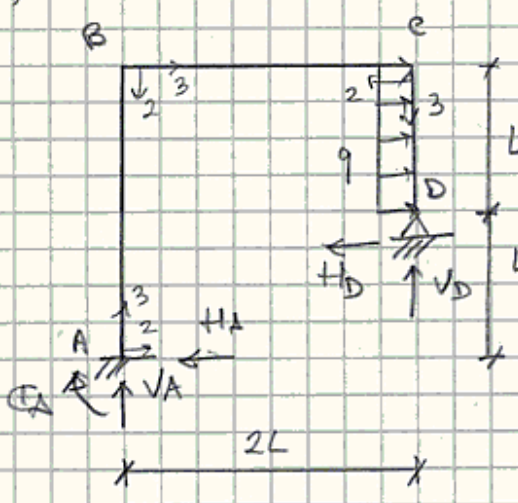
CORSO DI LAUREA IN INGEGNERIA MECCANICA
UNIVERSITÀ DEGLI STUDI DI FERRARA
PROVA SCRITTA DI STATICA
FERRARA, 1/7/2015



l = 2 m, h = 1m, q = 20 kN/m,
E = 210 GPa, σ_{AMM}= 240 MPa

1. Utilizzando il metodo delle forze risolvere la travatura in presenza del carico q e disegnare i diagrammi delle caratteristiche della sollecitazione (N, T, M).
2. Progettare la travatura con profilati IPE.
3. Calcolare lo spostamento orizzontale del piano superiore BC (ipotizzare trascurabili le deformazioni assiali).
4. Risolvere nuovamente la travatura considerando, in aggiunta al carico q, un abbassamento di 1 cm del vincolo in D.

1)



$$H_A + H_D - qL = 0$$

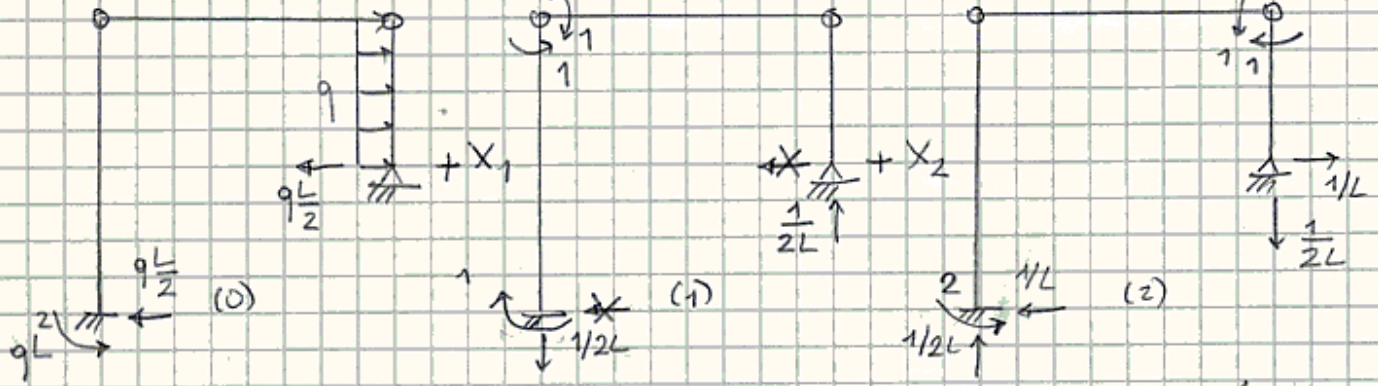
$$V_A + V_D = 0$$

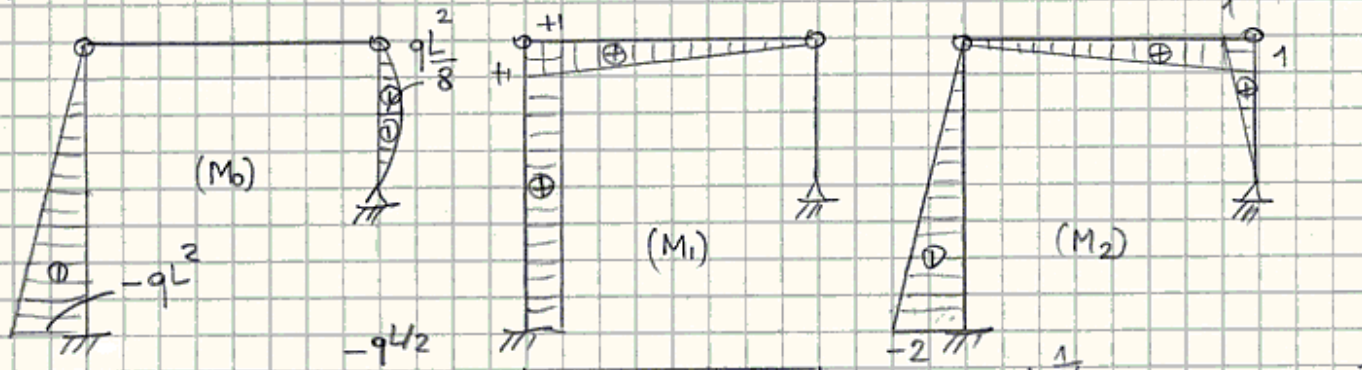
$$(A) \quad H_D L + V_D 2L - qL\frac{3}{2}L - C_A = 0$$

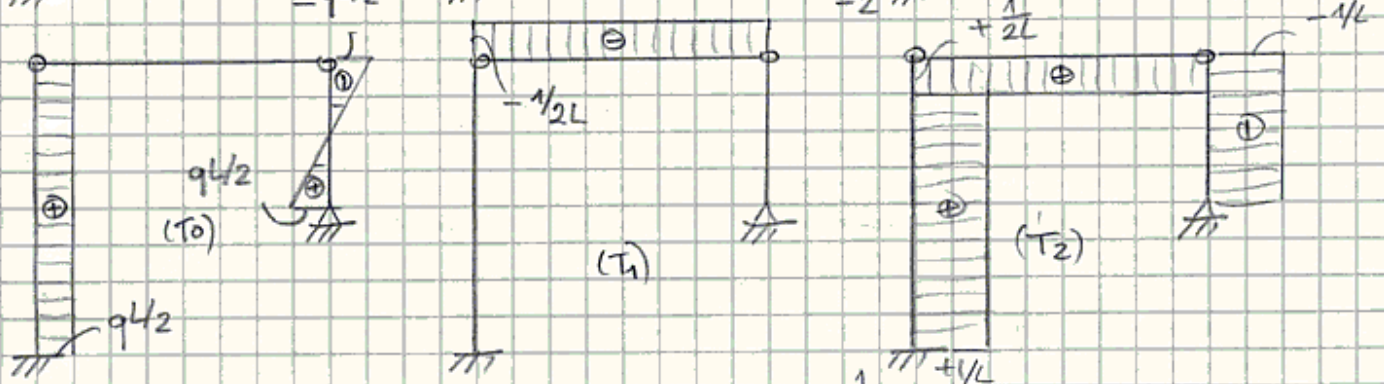
La travatura è due volte iperstatica

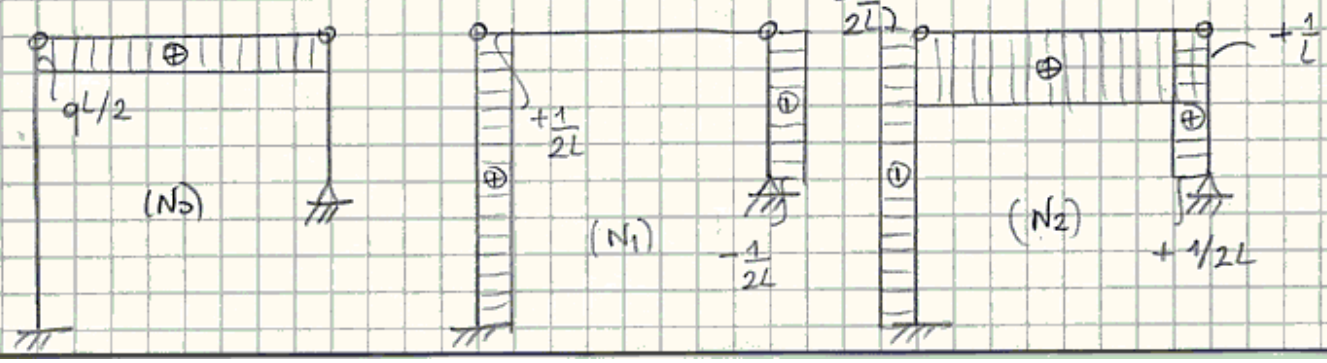
$X_1 = M_B$

$X_2 = M_C$









$$EI_1 \delta_{10} = 2L\left(-qL^2\right)\frac{1}{2}\cdot 1 = -qL^3$$

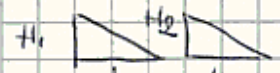
$$EI_1 \delta_{20} = \frac{1}{3}2L\left(-qL^2\right)(-2) - \frac{1}{24}qL^3 = \frac{31}{24}qL^3$$

$$EI_1 \delta_{11} = 2L + \frac{1}{3}2L = \frac{4}{3}\cdot 2L = \frac{8}{3}L$$

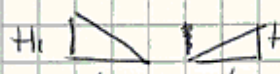
$$EI_1 \delta_{12} = -2\cdot 2L\,\frac{1}{2} + \frac{1}{6}2L = -\frac{5}{3}L$$

$$EI_1 \delta_{22} = \frac{1}{3}4\cdot 2L + \frac{1}{3}2L + \frac{1}{3}L = \frac{11}{3}L$$

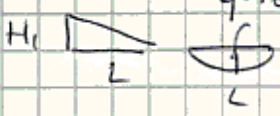
$$\frac{6\cdot 8}{3\cdot 8} - \frac{1}{24} = \frac{31}{24}$$



$$\frac{1}{3}H_1H_2L$$



$$\frac{1}{6}H_1H_2L$$



$$\frac{1}{24}H_1qL^3$$

$$\begin{bmatrix} 8/3 & -5/3 \\ -5/3 & 11/3 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = 3qL^2\begin{bmatrix} 1 \\ -\frac{31}{24} \end{bmatrix}$$

$$24\cdot 11 - 5\cdot 31 = 109$$

$$5 - \frac{31}{3} = \frac{15-31}{3} = -\frac{16}{3}$$

$$\det = 88 - 25 = 63$$

$$X_1 = \frac{3qL^2}{63}\det\begin{bmatrix} 1 & -5 \\ -\frac{31}{24} & 11 \end{bmatrix} = \frac{3qL^2}{63}\left(11 - \frac{31\cdot 5}{24_8}\right) = \frac{109}{504}qL^2 = 4,33\ kN\,m$$

$$X_2 = \frac{3qL^2}{63}\det\begin{bmatrix} 8 & 1 \\ -5 & -\frac{31}{24} \end{bmatrix} = \frac{3qL^2}{63}\left(-\frac{31}{3}+5\right) = -\frac{16}{63}qL^2 = -5,08\ kN\,m$$

Diagrammi della pagina seguente.

Calcoli:

$$M_A = -qL^2 + X_1 - 2X_2 = -qL^2 + \frac{109}{504}qL^2 + \frac{32}{63}qL^2 = \frac{-504+109+32\cdot 8}{63\cdot 8}qL^2 = -\frac{139}{504}qL^2$$

$$= -5,51\ kN\,m$$

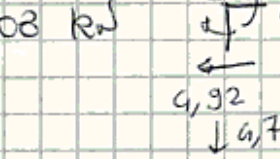
$$T_A = q\frac{L}{2} + \frac{X_2}{L} = q\frac{L}{2} - \frac{16}{63}qL = \frac{63-32}{63\cdot 2}qL = +\frac{31}{126}qL = +4,92\ kN$$

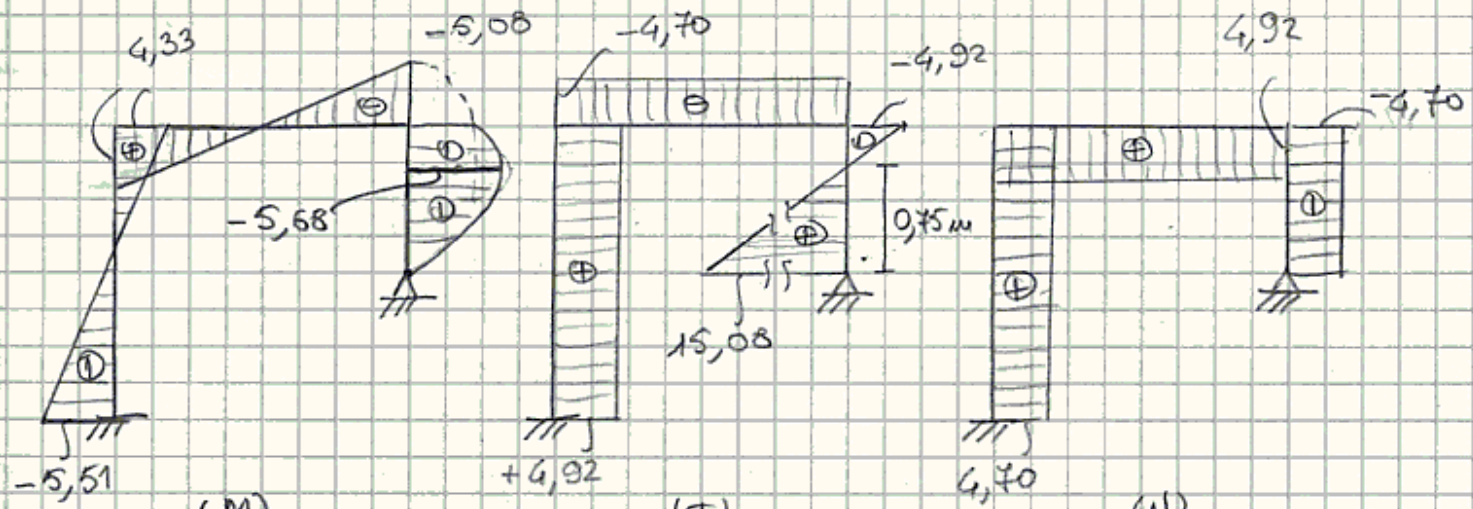
$$T_{B_0}^{+} = -\frac{X_1}{2L} + \frac{X_2}{2L} = -\frac{qL}{2}\left(\frac{109}{504} + \frac{16}{63}\right) = -\frac{79}{336}qL = -4,70\ kN$$

$$T_{C_0}^{+} = -q\frac{L}{2} - \frac{X_2}{L} = -qL\left(\frac{1}{2} - \frac{16}{63}\right) = -\frac{31}{126}qL = -4,92\ kN$$

$$T_D = q\frac{L}{2} - \frac{X_2}{L} = +qL\left(\frac{1}{2} + \frac{16}{63}\right) = \frac{95}{126}qL = 15,08\ kN$$

$$M_{MAX} = 15,08\cdot 0,75 + 20,0\frac{(0,75)^2}{2} = 5,68\ kN\,m$$

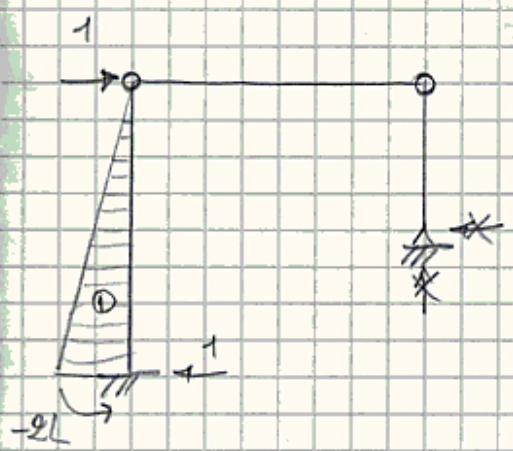




2) Progetto: $W_1 \geq \frac{M_{max}}{\sigma_{AMM}} = \frac{5,68 \cdot 10^3}{240 \cdot 10^6} \cdot 10^6 \, cm^3 = 23,66 \, cm^3$

IPE 100 $\begin{cases} W_1 = 34,20 \, cm^3 \\ I_1 = 171 \, cm^4 \\ A = 10,32 \, cm^2 \end{cases}$

3) Spostamento:



$$1 \cdot \delta_B = \frac{1}{EI_1} \int_{sist} MM^* \, dz$$

$$= \frac{1}{EI_1} \int_0^{2L} -z \cdot \left(\frac{109}{504} qL^2 - \frac{31}{126} qLz \right) dz$$

$$= \frac{1}{EI_1} \left[\frac{109}{504} qL^2 \frac{z^2}{2} + \frac{31}{126} qL \frac{z^3}{3} \right]_0^{2L}$$

$$= -\frac{qL^4}{EI_1} \left[\frac{109 \cdot 4}{1008} - \frac{31 \cdot 8}{378} \right] = + \frac{169}{756} \frac{qL^4}{EI_1}$$

$$= + \frac{169}{756} \frac{20 \cdot 10^3 \cdot 10^2}{210 \cdot 10^9 \, 171 \cdot 10^{-8}} = 1,24 \, cm$$

Cedimento in D: $\Delta = 1\,cm$

$$y_{10} + y_{11} X_1 + y_{12} X_2 = y_1$$
$$y_{20} + y_{12} X_1 + y_{22} X_2 = y_2$$

$$y_1 = -\frac{\Delta}{2L}$$

$$y_2 = \frac{\Delta}{2L}$$

$$\begin{bmatrix} 8/3 & -5/3 \\ -5/3 & 11/3 \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = 3qL^2 \begin{bmatrix} 1 \\ -\frac{31}{24} \end{bmatrix} + \frac{3\Delta}{2L} EI_1 \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Calcolo il contributo dovuto a Δ:

$$\begin{cases} X_1^{\Delta} = \dfrac{3EI_1\Delta}{2L} \dfrac{1}{63} \det \begin{bmatrix} -1 & -5 \\ 1 & 11 \end{bmatrix} = \dfrac{3EI_1\Delta}{126L}(-11+5) = -\dfrac{6EI_1\Delta}{126L} \\ X_2^{\Delta} = \dfrac{3EI_1\Delta}{2L} \dfrac{1}{63} \det \begin{bmatrix} 8 & -1 \\ -5 & 1 \end{bmatrix} = \dfrac{3EI_1\Delta}{126L}(8-5) = \dfrac{9EI_1\Delta}{126L} \end{cases}$$

In totale (q e Δ) si ha che:

$$\begin{cases} X_1 = \dfrac{109}{504} qL^2 - \dfrac{6EI_1\Delta}{126L} = (4,33 - 0,17)\,kNm = 4,16\,kNm \\ X_2 = -\dfrac{16}{63} qL^2 + \dfrac{9EI_1\Delta}{126L} = (-5,08 + 0,25)\,kNm = -4,83\,kNm \end{cases}$$

Calcoli:

$$\frac{150EI_1\Delta}{126L} = \frac{150 \cdot 210 \cdot 10^{9} \cdot 171 \cdot 10^{-8} \cdot 10^{-2}}{126}\, 10^{-3}\, kNm = 0,176\,kNm$$

$$\frac{9EI_1\Delta}{126L} = \frac{9 \cdot 210 \cdot 10^{9} \cdot 171 \cdot 10^{-8}\, 10^{-2}}{126}\, 10^{-3}\, kNm = 0,25\,kNm$$