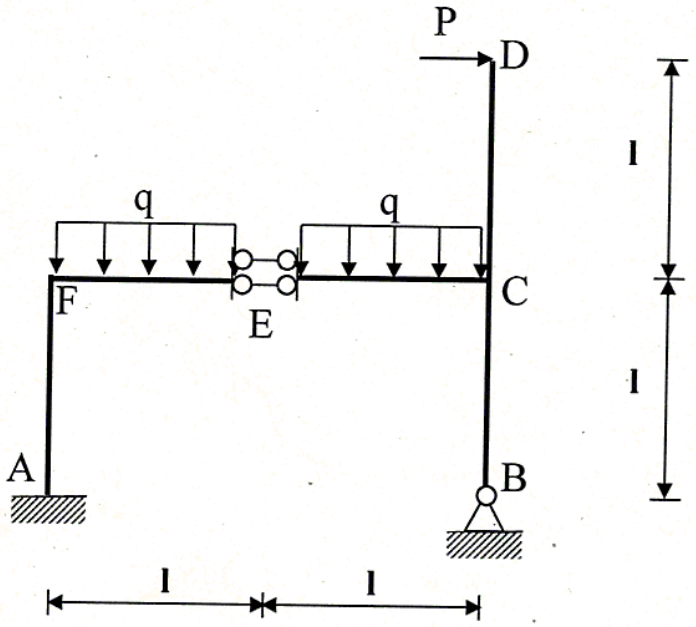
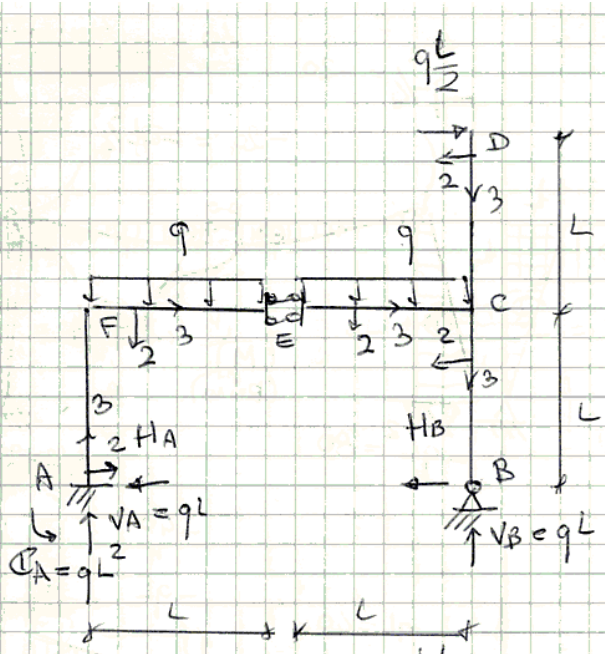
CORSO DI LAUREA IN INGEGNERIA MECCANICA
UNIVERSITÀ DI FERRARA
PROVA SCRITTA DI STATICA
10/02/2016



l= 2 m, q = 20 kN/m, P = 20 kN,
E = 210 GPa, σ_{amm} = 240 MPa

La travatura in figura deve essere realizzata con profilati IPE.

- Disegnare i diagrammi quotati delle caratteristiche della sollecitazione.
- Dimensionare la travatura.
- Calcolare lo spostamento orizzontale del nodo D.
- Disegnare i diagrammi quotati delle caratteristiche della sollecitazione in presenza di un cedimento orizzontale verso destra pari a 2 cm del vincolo in B.



$(\rightarrow)\quad H_A + H_B = q\frac{L}{2}$

$(\uparrow)\quad V_A + V_B = 2qL$

$(A\uparrow)\quad V_B 2L - 2qL\cdot L - q\frac{L}{2}\cdot\frac{3}{2}L + C_A = 0$

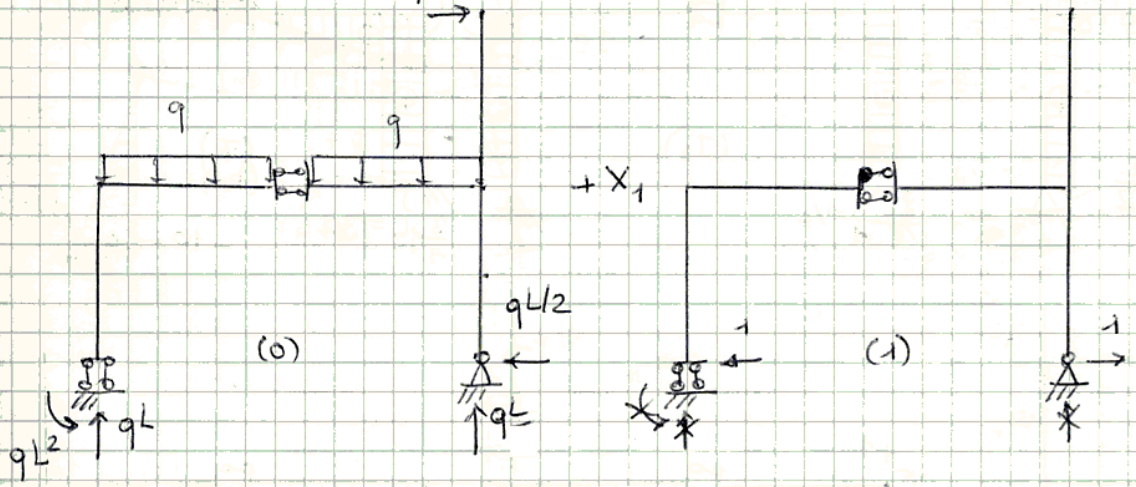
Eq.ne della deformazione in E:

$0 = T_E^+ = qL - V_B \rightarrow V_B = qL = V_A$

$\hookrightarrow C_A = 3qL^2 - 2V_B = qL^2$

La struttura è 1 volta iperstatica

Incognita iperstatica: $X_1 = H_A$.



Diagrammi parziali della pagina seguente

$$EI_1 y_{10} = -qL^2\frac{L^2}{2} - L\left(q\frac{L}{2}^2 2L + 2\frac{1}{3}L\,q\frac{L}{2}^2\right) - \frac{1}{3}LL\,q\frac{L}{2}^2$$

$$= -qL^4\left(\frac{1}{2} + 2 + \frac{2}{3} + \frac{1}{3}\right) = -\frac{4}{2}qL^4 = -2qL^4$$

$$EI_1 y_{11} = 2\cdot\frac{1}{3}LL^2 + 2LL^2 = \frac{8}{3}L^3$$

$$X_1 = 2\cdot\frac{3}{8}qL = \frac{3}{4}qL = 30\ kN$$

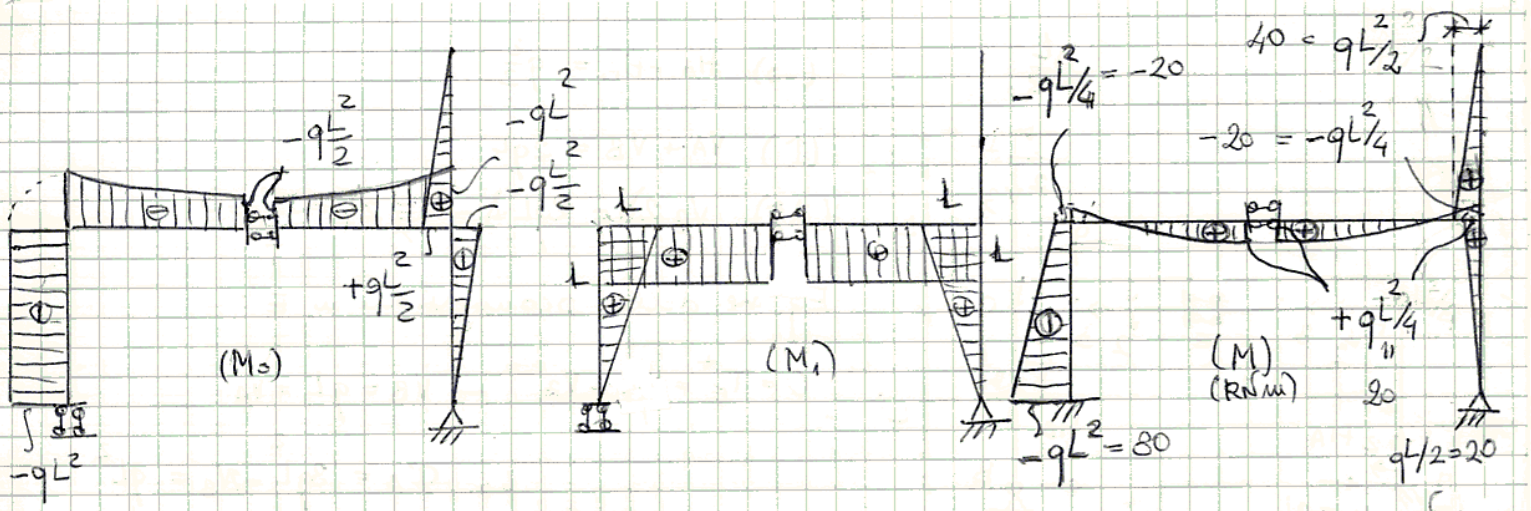
$$H_B = q\frac{L}{2} - \frac{3}{4}qL = -q\frac{L}{4} = -10\ kN$$

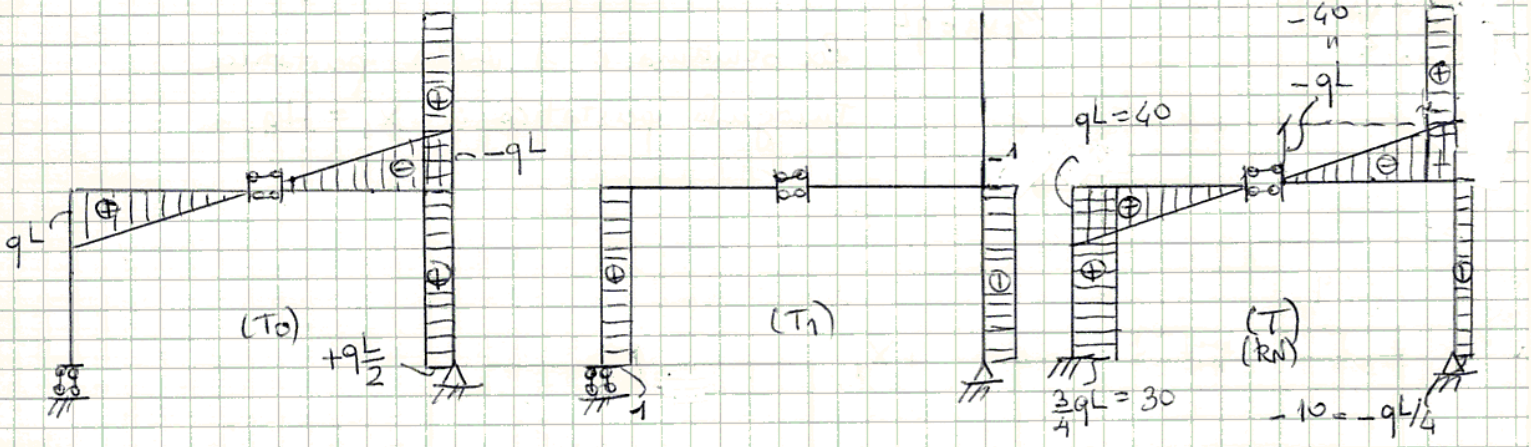
I diagrammi finali sono riportati alla pagina seguente

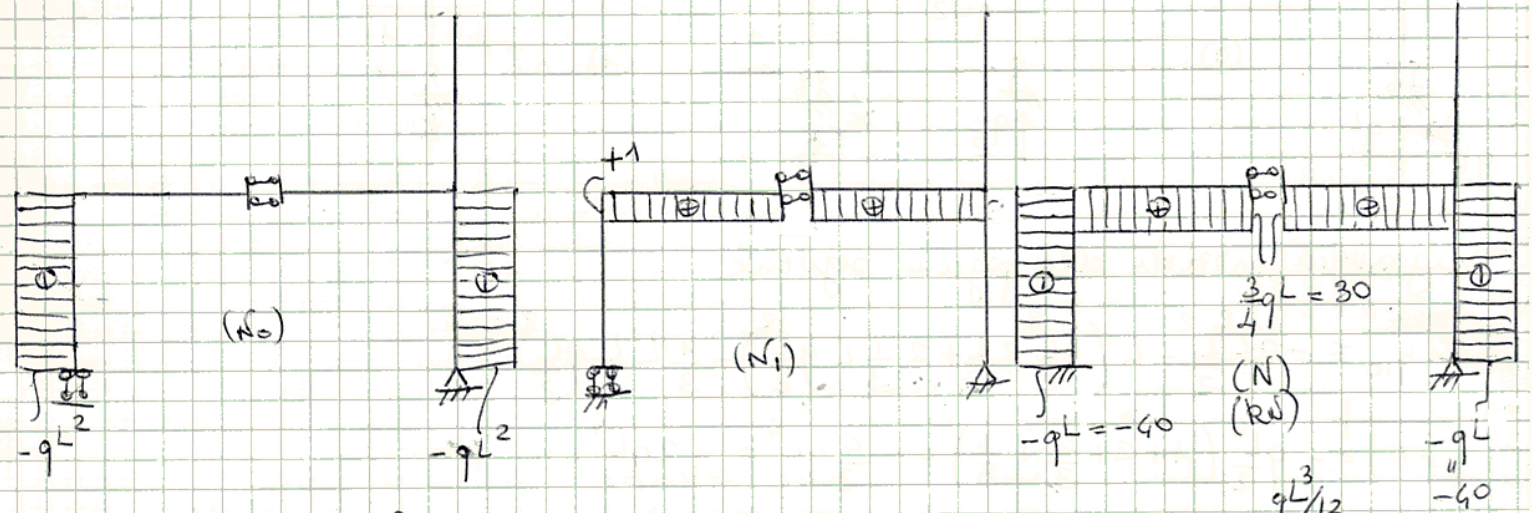
Dimensionamento:

$$W \geq \frac{M_{max}}{\sigma_{AMM}} = \frac{qL^2}{\sigma_{AMM}} = \frac{80\cdot 10^3}{240\cdot 10^6/3}\ m^3 = \frac{10^3}{3}\ cm^3 = 334\ cm^3 \rightarrow$$

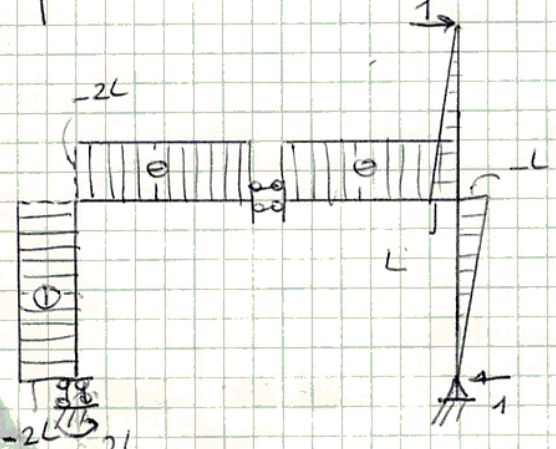
IPE 270 $\begin{cases} I_1 = 5790\ cm^4 \\ W_1 = 420\ cm^3 \end{cases}$







Spostamento orizzontale in D:



$$\delta = \frac{1}{EI_1}\left\{2L\left(qL^2+q\frac{L^2}{4}\right)\frac{L}{2} + (-2L)\,2\int_0^L\left(-q\frac{x^2}{2}+q\frac{L^2}{4}\right)dx + \frac{1}{3}LLqL^2 - \frac{1}{3}LLq\frac{L^2}{4}\right\}$$

$$= \frac{1}{EI_1}qL^4\left\{\frac{5}{4} - \frac{4}{12} + \frac{1}{6} - \frac{1}{12}\right\} = \frac{1}{EI_1}qL^4\frac{(15-5+2)}{12}$$

$$= \frac{qL^4}{EI_1}$$

$$= \frac{20\cdot 10^3 \cdot 16\cdot 10^2}{21\cdot 10^6 \cdot 5790\cdot 10^{-8}}\ \text{cm}$$

$$= 2{,}63\ \text{cm}$$

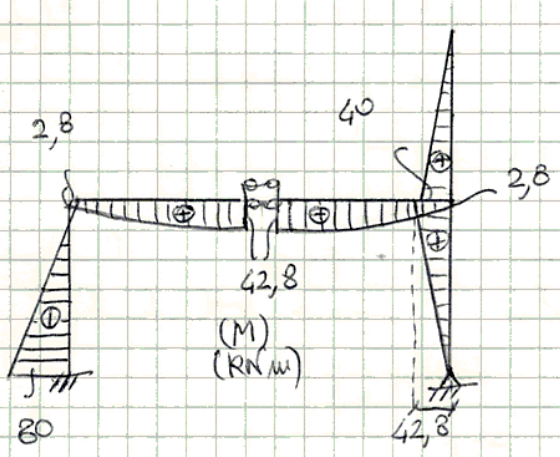
Cedimento:

$\eta_1 = 1 \cdot \delta$

$\eta_{10} + \eta_{11} X_1 = \eta_1$

$$X_1 = -\frac{\eta_{10}}{\eta_{11}} + \frac{\eta_1}{\eta_{11}} = \frac{3}{4} qL + \frac{3EI_1\delta}{8L^3} = \left(30 + \frac{3 \cdot 210 \cdot 10^8 \cdot 5792 \cdot 10^{-8} \cdot 0{,}02}{8 \cdot 8 \cdot 10^3}\right) kN$$

$$\overset{!}{=} (30 + 11{,}4)\, kN = 41{,}4\, kN$$



Calcoli

$qL^2 = 20 \cdot 4\ kNm = 80\ kNm$

$-80 + 2 \cdot 41{,}4 = 2{,}8\ kNm$

$-40 + 2 \cdot 41{,}4 = 42{,}8\ kNm$