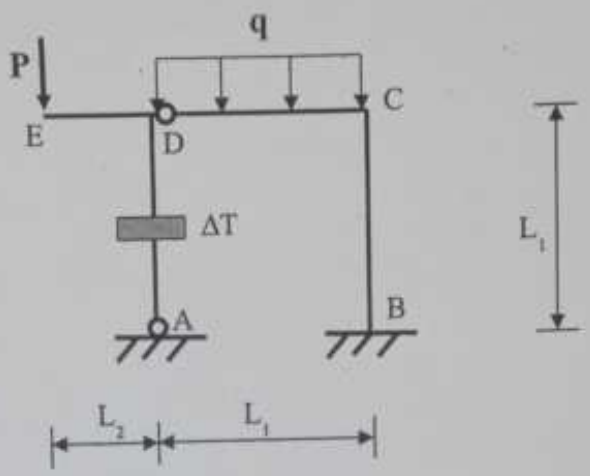
CORSO DI LAUREA IN INGEGNERIA MECCANICA
UNIVERSITÀ DI FERRARA
PROVA SCRITTA DI STATICA
13/1/2015

A



$L_1 = 5$ m, $L_2 = 2.5$ m, $q = 20$ kN/m, $P = 50$ kN

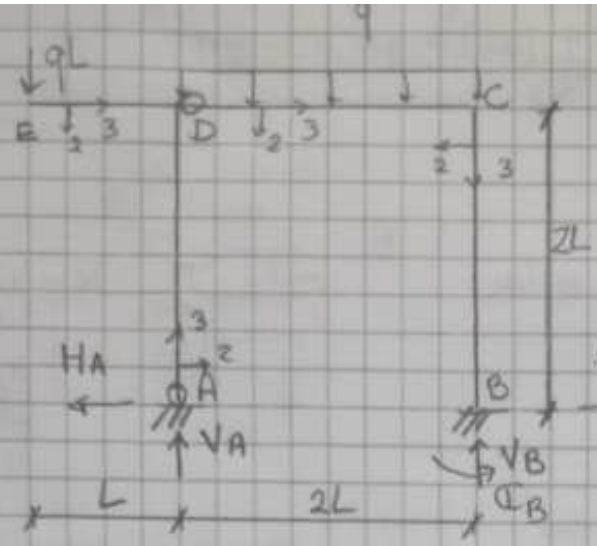
$\sigma_{AMM} = 240$ MPa, $E = 210$ GPa

$\Delta T = +10°C$, $\alpha = 10^{-5}\ °C^{-1}$

La travatura in figura deve essere realizzata con profilati IPE.

- Disegnare i diagrammi quotati delle caratteristiche della sollecitazione in sola presenza dei carichi q e P.
- Dimensionare la travatura.
- Calcolare lo spostamento verticale del punto E.
- Disegnare nuovamente i diagrammi quotati considerando, in aggiunta ai carichi q e P, anche il riscaldamento del tratto AD.

A)



$L = 2{,}5\ m$

$q = 20\ kN/m$

$P = 50\ kN = qL$

$qL^2 = 125\ kNm$

$(D)_{ADE}$ $H_A 2L = qL^2 \leftrightarrow H_A = qL/2$

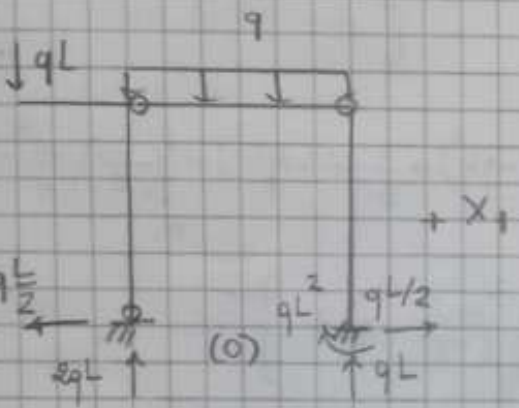
H_B $(\rightarrow)$ $H_B = H_A$

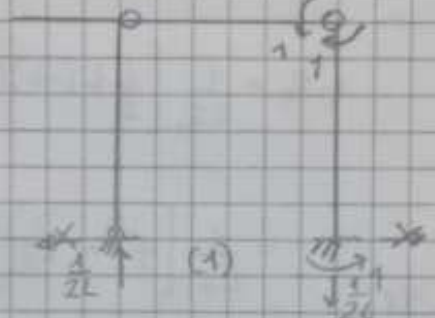
$(\uparrow)$ $V_A + V_B = 3qL$

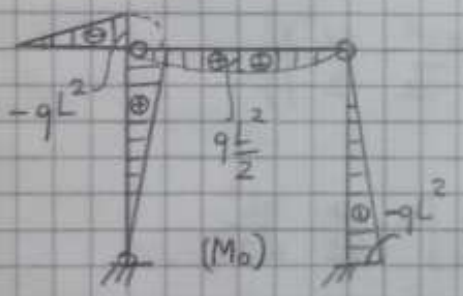
(A) $M_B + V_B 2L - 2qL^2 + q\frac{L^2}{2} = 0$

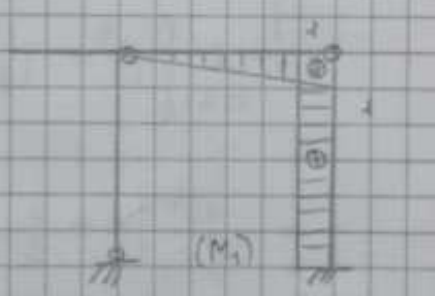
↳ Trave una volta iperstatica.

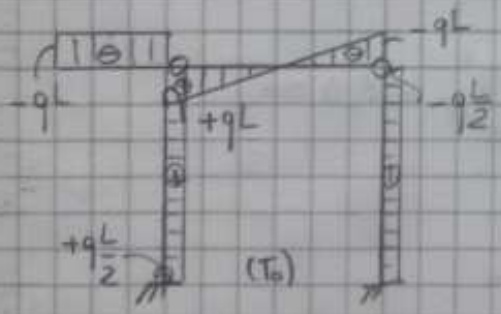
$X_1 = M_C$

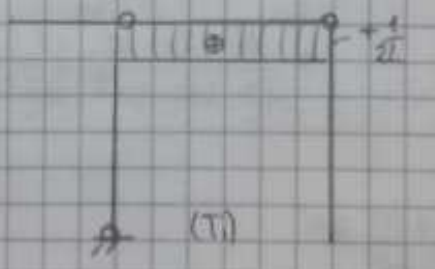


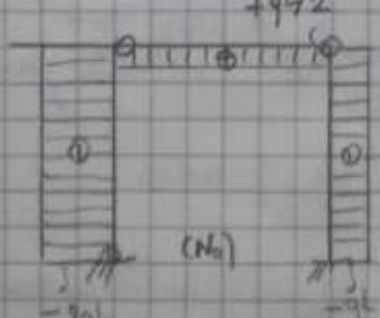


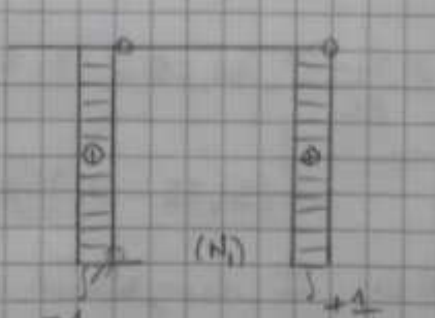


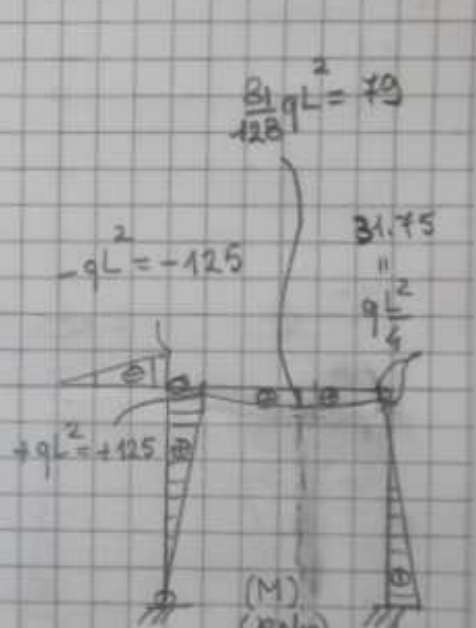


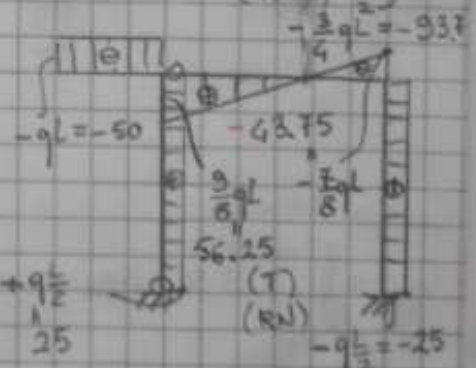


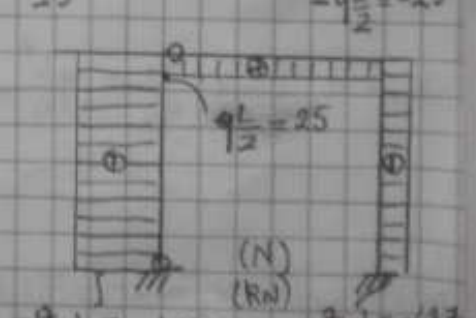








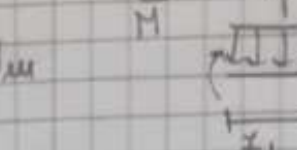




$$EI_1 \eta_{10} = \int_0^{2L} \frac{x_3}{2}\left(qL - q\frac{x_3}{2}\right) dx_3 - \frac{qL^3}{8} \cdot \frac{1}{2} = \frac{q}{2}\left(\frac{1}{3}L\right)^3 - qL^2 = -\frac{2}{3}qL^2$$

$$EI_1 \eta_{11} = \frac{1}{3}2L + 2L = \frac{8}{3}L$$

$$X_1 = -\frac{\eta_{10}}{\eta_{11}} = \frac{2}{8} qL^3 \frac{3}{8L} = \frac{qL^2}{4} = 31.75 \text{ kNm}$$



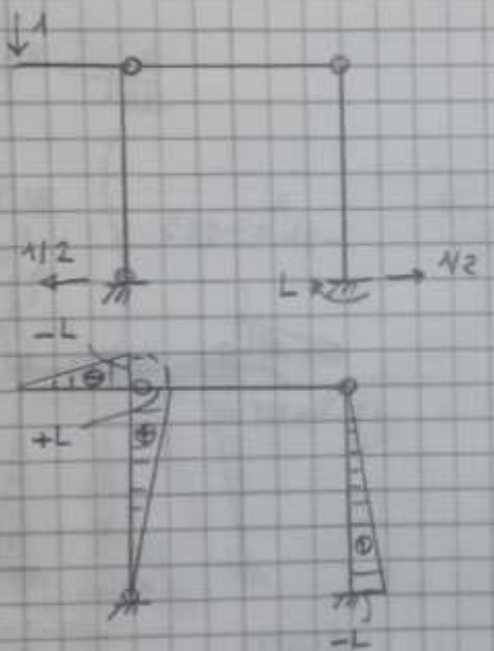
$$\overline{M} = \left(+\frac{1}{4} + \frac{1}{2}\frac{1}{64}\right) qL^2 = \frac{81}{128} qL^2$$

Dimensionamento:

$$W_1 \geq \frac{qL^2}{6\sigma_{amm}} = \frac{125 \cdot 10^2}{240 \cdot 10^6} \cdot 10^6 \text{ cm}^3 = 520.8 \text{ cm}^3$$

IPE 300 $\begin{cases} W_1 = 557,1 \text{ cm}^3 \\ I_1 = 8356 \text{ cm}^4 \\ A = 53,81 \text{ cm}^2 \end{cases}$

Spostamento verticale in E:



$$1 \cdot v_E = \frac{1}{EI_1}\left[\frac{1}{3}L(-L)(-qL^2) + \frac{1}{3}2L(qL^2) + \int_0^{2L}\left(-\frac{x_3}{2}\right)\left(\frac{qL^2}{4} - \frac{qL}{4}x_3\right) dx_3\right]$$

$$= \frac{1}{EI_1}\left[qL^4 + q\frac{L^2}{8}\int_0^{2L}\left(\frac{2x_3^2}{L} - x_3\right) dx_3\right]$$

$$= \frac{1}{EI_1}\left\{qL^4 + q\frac{L^2}{8}\left[\frac{16L^3}{3L} - \frac{4L^2}{2}\right]\right\}$$

$$= \frac{qL^4}{EI_1}\left[1 + \frac{2}{3} - \frac{1}{4}\right] = \frac{17}{12}\frac{qL^4}{EI_1}$$

$$= \frac{17 \cdot 20 \cdot 10^6 \cdot (2,5)^4 \cdot 10^2}{12 \cdot 200 \cdot 10^9 \cdot 8356 \cdot 10^{-8}} = 6,3 \text{ cm}$$

Carico termico:

$$\eta_{1t} = \int_{AD} \alpha \Delta T N_1 \, dx_3 = -\frac{\alpha \Delta T}{2L} 2L$$

$$X_1 = -\frac{\eta_{10}}{\eta_{11}} + \frac{\eta_{1t}}{\eta_{11}} = \frac{qL^2}{4} - \frac{3EI_1 \alpha \Delta T}{8L} = \left(31,75 - \frac{3 \cdot 210 \cdot 10^3 \cdot 8356 \cdot 10^{-8} \cdot 10^{-5} \cdot 10 \cdot 10^3}{8 \cdot 2,5}\right) \text{kNm}$$

$$= (31,75 - 0,26) \text{ kNm} = 31,49 \text{ kNm}$$

$M_B = -qL^2 + X_1 = -93,51$ kNm. I diagrammi restano sostanzialmente invariati.

A.2

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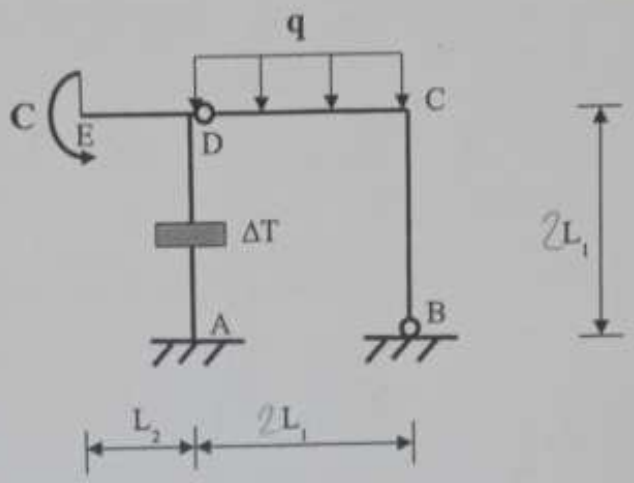
B

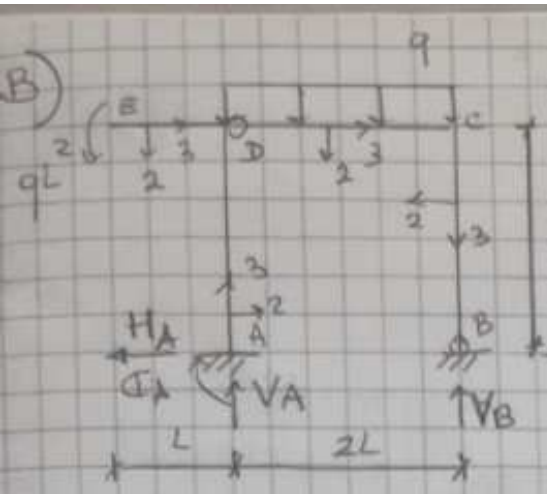


$L_1 = 4$ m, $L_2 = 2$ m, $q = 25$ kN/m, $C = 100$ kNm
$\sigma_{AMM} = 240$ MPa, $E = 210$ GPa
$\Delta T = -10°C$, $\alpha = 10^{-5}\ °C^{-1}$

La travatura in figura deve essere realizzata con profilati IPE.

- Disegnare i diagrammi quotati delle caratteristiche della sollecitazione in sola presenza dei carichi q e C.
- Dimensionare la travatura.
- Calcolare lo spostamento verticale del punto E.
- Disegnare nuovamente i diagrammi quotati considerando, in aggiunta ai carichi q e C, anche il raffreddamento del tratto AD.

B)



$(\rightarrow)\ H_A = H_B$

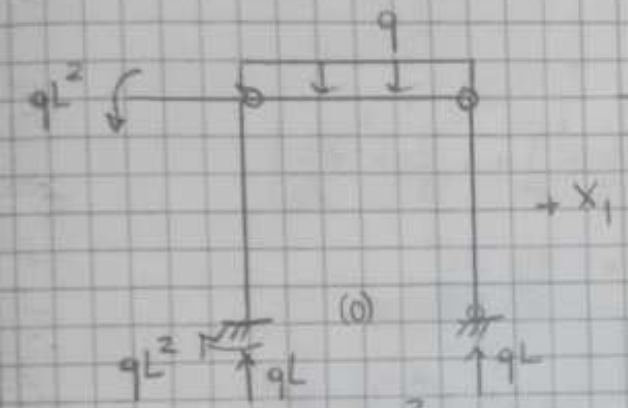
$(\uparrow)\ V_A + V_B = 2qL$

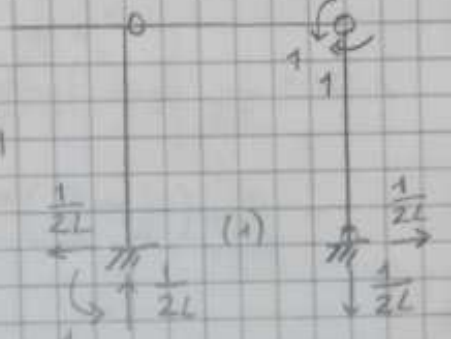
$(\curvearrowleft)\ V_B\,2L - 2qL^2 - \mathcal{M}_A + qL^2 = 0$

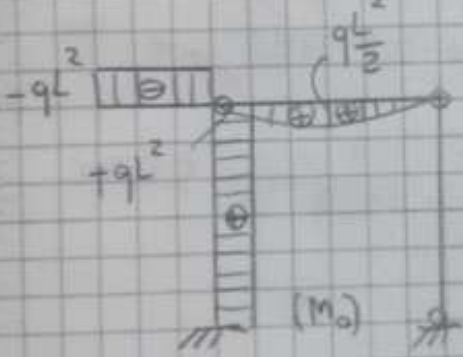
$(D\curvearrowleft)_{ADE}\ \mathcal{M}_A + H_A\,2L = qL^2$

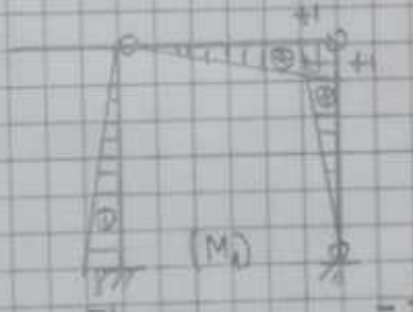
Izaratuna una volta iperstatica.

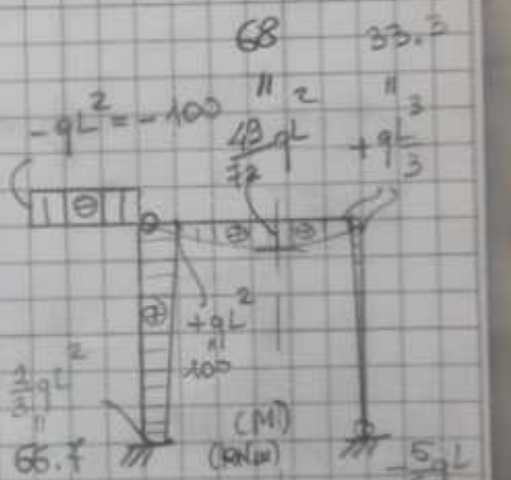
$X_1 = M_C$

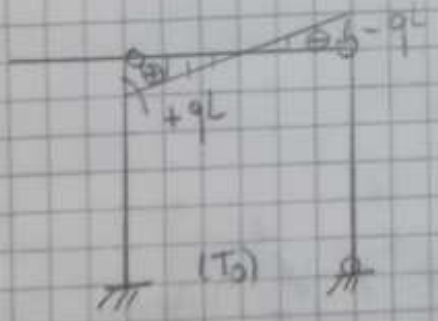


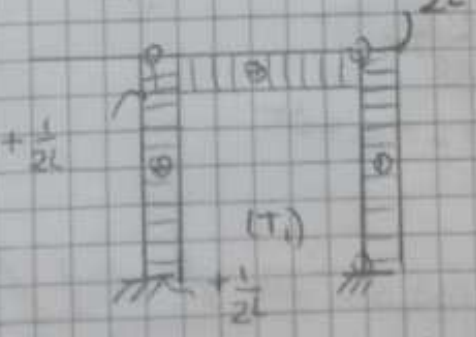


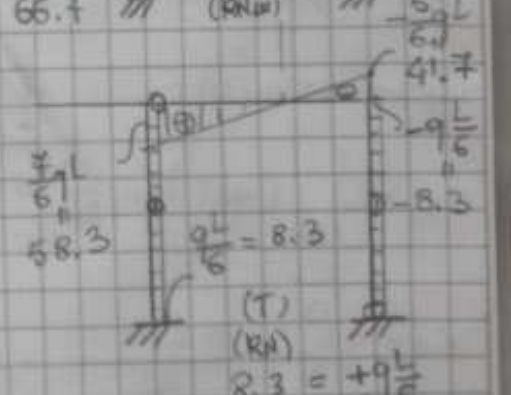


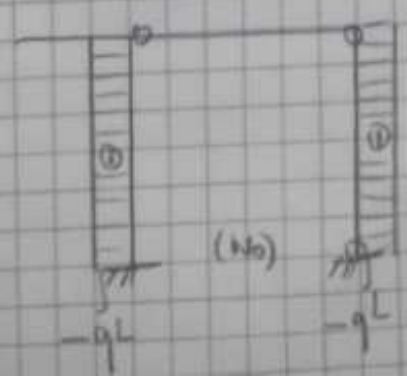


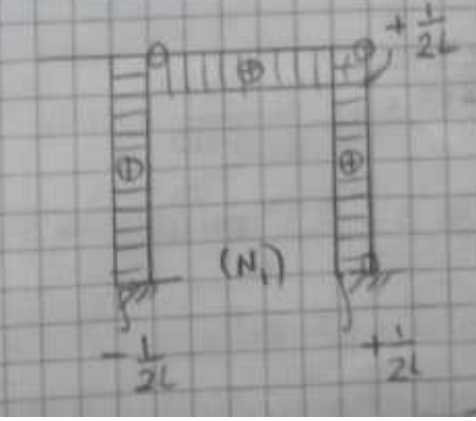


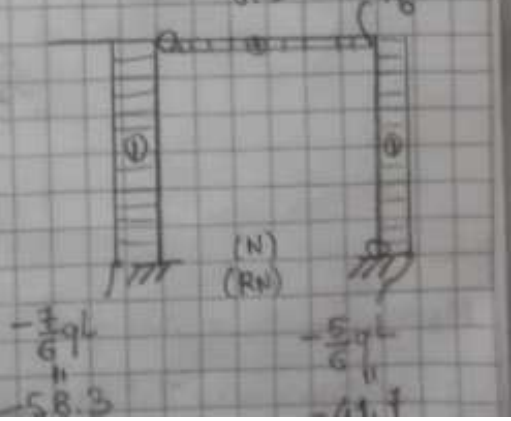












$$EI_1 \varphi_{10} = qL^2 \frac{1}{2}(-1)2L + \frac{q(2L)^2}{24} = -qL^3 + \frac{qL^3}{3} = -\frac{2}{3}qL^3$$

$$EI_1 \varphi_{11} = \frac{1}{2} \cdot \frac{1}{3} 2L \cdot 1 = 2L$$

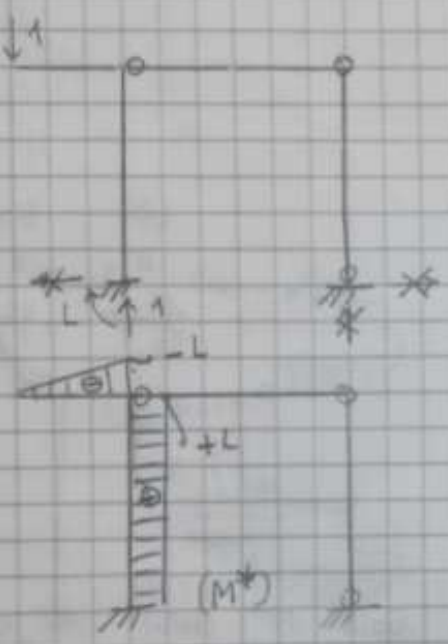
$$X_1 = \frac{3}{2} \frac{qL}{2} = \frac{qL^2}{3} = 33{,}3 \text{ kN m}$$

Dimensionamento:

$$W_1 \geq \frac{qL^2}{6A\sigma_{amm}} = \frac{100 \cdot 10^2}{240 \cdot 10^6} \cdot 10^6 = 416{,}7 \text{ cm}^3$$

IPE 270 $\begin{cases} W_1 = 428{,}9 \text{ cm}^3 \\ I_1 = 5790 \text{ cm}^4 \\ A = 45{,}95 \text{ cm}^2 \end{cases}$

Spostamento verticale in E:



$$1 \cdot v_E = \frac{1}{EI_1}\left\{(-qL^2)\left(-\frac{1}{2}L^2\right) + L\,\frac{5}{3}qL^2\,\frac{L}{2}\right\}$$

$$= \frac{qL^4}{EI_1}\left(\frac{1}{2} + \frac{5}{3}\right) = \frac{13}{6}\frac{qL^4}{EI_1}$$

$$= \frac{13 \cdot 2{,}5 \cdot 10^3 \cdot 16 \cdot 10^4}{6 \cdot 210 \cdot 10^9 \cdot 5790 \cdot 10^{-8}} \text{ cm}$$

$$= \frac{13 \cdot 25 \cdot 10^4 \cdot 16}{6 \cdot 21 \cdot 5790} = 7{,}1 \text{ cm}$$

Carico termico:

$$M_{1t} = \int_{AD} (-\alpha\Delta T)\left(-\frac{1}{2L}\right) dx_3 = +\alpha\Delta T$$

$$X_1 = \frac{qL^3}{3} - \frac{EI_1 \alpha \Delta T}{2L} = \left(33{,}3 - \frac{210 \cdot 10^6 \cdot 5790 \cdot 10^{-8} \cdot 10^{-5} \cdot 10^4}{4}\right) \text{kN m}$$

$$= \left(33{,}3 - \frac{21 \cdot 579}{4 \cdot 10\,000}\right) \text{kN m}$$

$$= (33{,}3 - 0{,}3)\,\text{kN m} = 33 \text{ kN m}$$

$$M_A = qL^2 - X_1 = (100 - 33)\,\text{kN m} = 67 \text{ kN m}$$

I diagrammi restano ovviamente invariati.

B.2