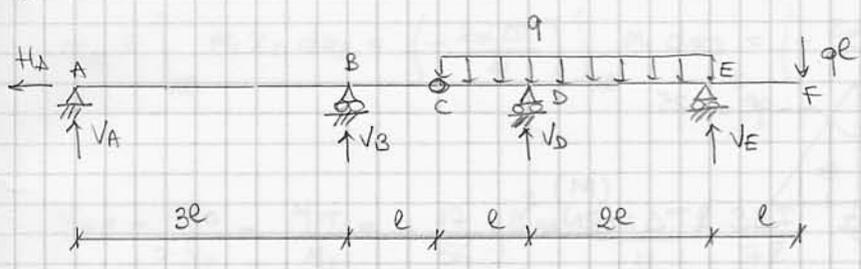


$$\begin{aligned}
 l_1 &= 3 \text{ m}, \quad l_2 = 1 \text{ m}, \quad l_3 = 2 \text{ m}, \\
 q &= 25 \text{ kN/m}, \quad P = 25 \text{ kN}, \\
 E &= 2.1 \cdot 10^3 \text{ kN/cm}^2, \quad \alpha = 10^{-5} \text{ } ^\circ\text{C}^{-1}, \quad \Delta T = 20 \text{ } ^\circ\text{C}
 \end{aligned}$$

La travatura iperstatica di figura è realizzata con profilati IPE 200 ($H = 200 \text{ mm}$, $A = 28.5 \text{ cm}^2$, $I_1 = 1943 \text{ cm}^4$).

1. Utilizzando il metodo delle forze risolvere la travatura in presenza dei soli carichi q e P e disegnare i diagrammi delle caratteristiche della sollecitazione (N , T , M).
2. Calcolare la rotazione del nodo E .
3. Risolvere nuovamente la travatura considerando anche il carico termico nel solo tratto AB e disegnare i diagrammi delle caratteristiche della sollecitazione (N , T , M) comprensivi sia di q , P che di ΔT .

c1)



$q = 25 \text{ kN/m}, \quad e = 1 \text{ m}$

$$\begin{cases} (\rightarrow) H_A = 0 \\ (\uparrow) V_A + V_B + V_D + V_E = 4ql \\ (C)_{ABC} V_A 4e + V_B e = 0 \\ (C)_{CDEF} V_D e + V_E 3e = 3ql \frac{3}{2} e + 4ql \frac{e^2}{2} \end{cases}$$

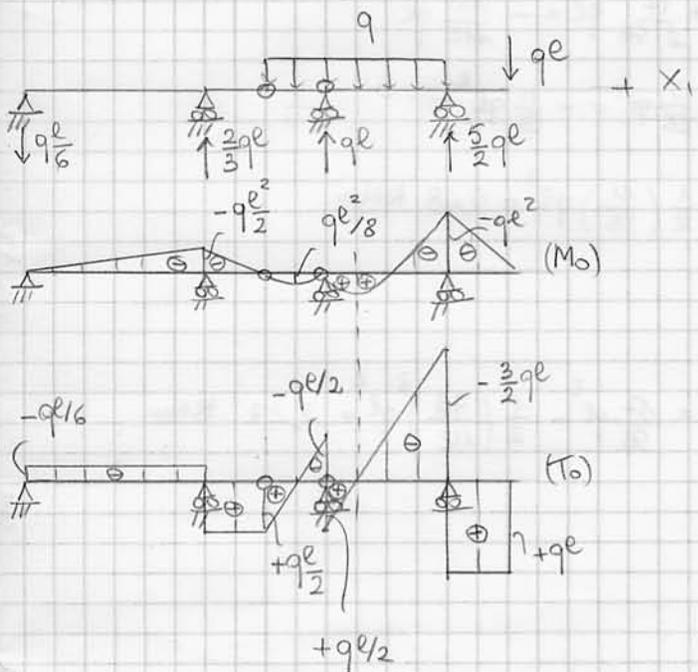
$$\begin{cases} H_A = 0 \\ V_B = -4V_A \\ V_D = \frac{17}{2} ql - 3V_E \\ V_A - 4V_A + \frac{17}{2} ql - 3V_E + V_E = 4ql \end{cases}$$

Travatura una volta iperstatica

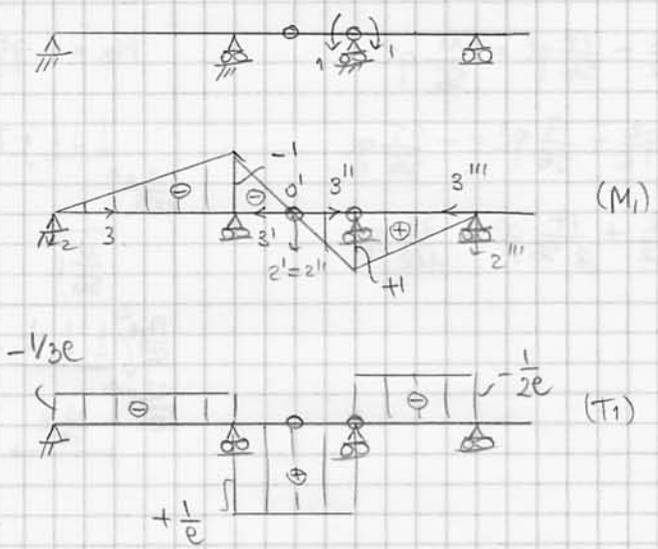
Incognita iperstatica: $X_1 = M_0$

$$\begin{cases} H_A = 0 \\ V_B = -4V_A \\ V_D = \frac{17}{2} ql - 3V_E = \frac{17}{2} ql - \frac{27}{4} ql + \frac{9}{2} V_A \\ V_E = \frac{1}{2} \left(\frac{17}{2} ql - 4ql - 3V_A \right) = \frac{9}{4} ql - \frac{3}{2} V_A \end{cases}$$

(0)



(1)

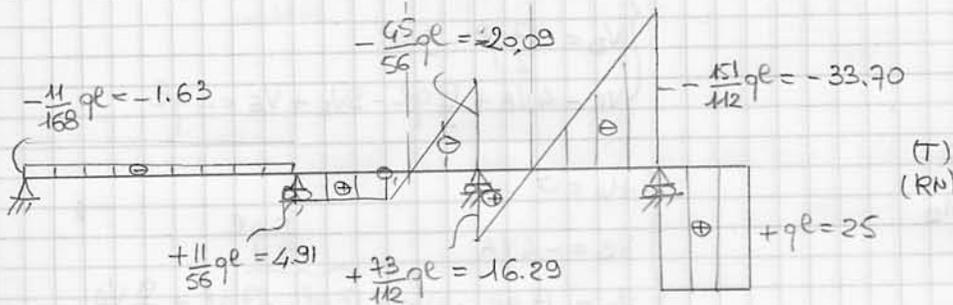
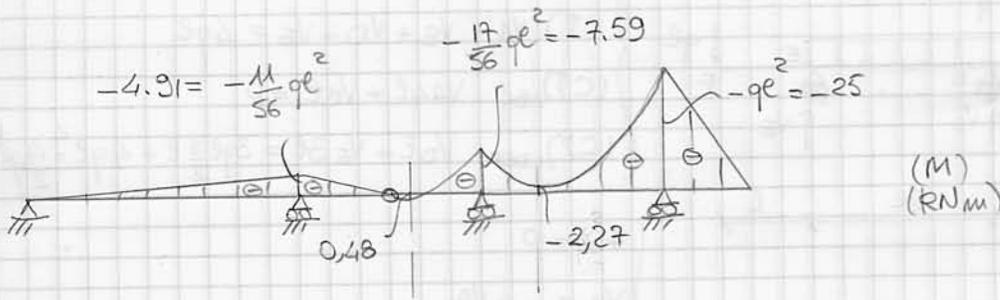


$$EI M_{10} = \int_0^{3e} \left(-\frac{ql}{6} x_3 \right) \left(-\frac{x_3}{3e} \right) dx_3 + \int_0^e \left(-\frac{ql}{2} x_3' \right) \left(-\frac{x_3'}{e} \right) dx_3' + \int_0^e \left(\frac{ql}{2} x_3'' - \frac{9}{2} x_3''^2 \right) \left(\frac{x_3''}{e} \right) dx_3'' + \int_0^{2e} \left(-ql^2 + \frac{3}{2} ql x_3''' - \frac{9}{2} x_3'''^2 \right) \left(\frac{x_3'''}{2e} \right) dx_3''' = \frac{ql^3}{2} + \frac{ql^3}{6} + \frac{ql^3}{24} + 0 = \frac{17}{24} ql^3$$

$$EI M_{11} = \int_0^{3e} \left(-\frac{x_3}{3e} \right)^2 dx_3 + 2 \int_0^e \left(-\frac{x_3'}{e} \right)^2 dx_3' + \int_0^{2e} \left(\frac{x_3'''}{2e} \right)^2 dx_3''' = \frac{1}{3} 3e + \frac{2}{3} e + \frac{1}{3} 2e = \frac{7}{3} e$$

$$X_1 = - \frac{M_{10}}{M_{11}} = - \frac{\frac{17}{24} ql^3}{\frac{7}{3} e} = - \frac{17}{56} ql^2 = -7,58 \text{ kNm}$$

Diagramm quotati:



Calcul:

$$T_A = -q \frac{l}{6} + \frac{17}{3 \cdot 56} q l = -\frac{11}{168} q l$$

$$T_B^+ = q \frac{l}{2} - \frac{17}{56} q l = \frac{11}{56} q l$$

$$T_D^- = -q \frac{l}{2} - \frac{17}{56} q l = -\frac{45}{56} q l$$

$$T_D^+ = q \frac{l}{2} + \frac{17}{2 \cdot 56} q l = \frac{73}{112} q l$$

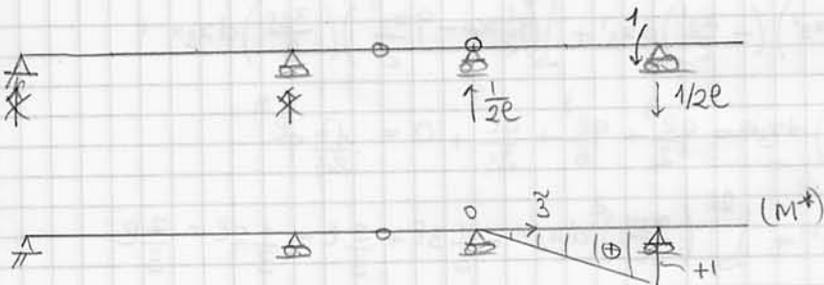
$$T_E = -\frac{3}{2} q l + \frac{17}{2 \cdot 56} q l = -\frac{151}{112} q l$$

$$M_B = -q \frac{l^2}{2} + \frac{17}{56} q l^2 = -\frac{11}{56} q l^2$$

$$\bar{M} = \frac{1}{2} \left(\frac{11}{56} \right) q l^2 = 0,48 \text{ kNm}$$

$$\bar{M} = \frac{17}{56} q l^2 - \frac{1}{2} \left(\frac{73}{112} \right)^2 q l^2 = 2,27 \text{ kNm}$$

c2)



$$1 \cdot \varphi_E = \frac{1}{EI_1} \int_0^{2l} \left(-\frac{17}{56} q l^2 + \frac{73}{112} q l x_3 - q \frac{x_3^2}{2} \right) \left(\frac{x_3}{2l} \right) dx_3 = -\frac{73}{168} \frac{q l^3}{EI_1} = -1,52^\circ$$

c3)

$$M_{1E} = \int_{AB} M_1 x_3 dx_3 = \left(-\frac{2\alpha\Delta T}{H} \right) \int_{AB} M_1 dx_3 = \left(-\frac{2\alpha\Delta T}{H} \right) \left(-\frac{3l}{2} \right) = \frac{3\alpha\Delta T l}{H}$$

$$X_1 = -\frac{M_{10}}{M_{11}} - \frac{M_{1T}}{M_{11}} = -\frac{17}{56} q l^2 - \frac{3\alpha\Delta T l}{H} \frac{3EI_1}{7l}$$

$$= -\frac{17}{56} q l^2 - \frac{9\alpha\Delta T EI_1}{7H} = (-7,58 - 0,52) \text{ KNm}$$

I diagrammi delle c.s. compresari da q , P e da ΔT non differiscono in modo significativo da quelli calcolati al punto (1).